



US Army Corps  
of Engineers

MISCELLANEOUS PAPER ITL-90-5

# REVIEW OF FINITE ELEMENT PROCEDURES FOR EARTH RETAINING STRUCTURES

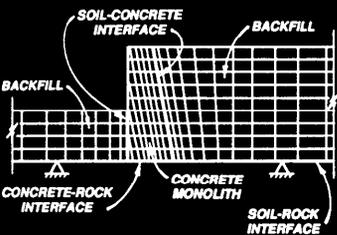
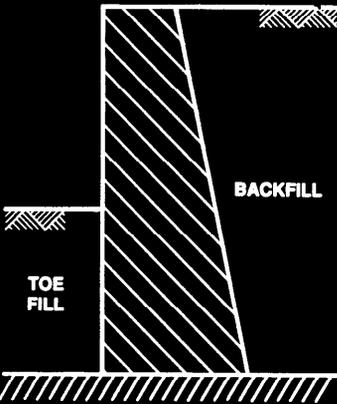
by

Robert M. Ebeling

Information Technology Laboratory

DEPARTMENT OF THE ARMY

Waterways Experiment Station, Corps of Engineers  
3909 Halls Ferry Road, Vicksburg, Mississippi 39180-6199



December 1990

Final Report

Approved for Public Release; Distribution Unlimited



Prepared for DEPARTMENT OF THE ARMY  
US Army Corps of Engineers  
Washington, DC 20314-1000

**REPORT DOCUMENTATION PAGE**

Form Approved  
 OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION Unclassified		1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release; distribution unlimited.	
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE			
4. PERFORMING ORGANIZATION REPORT NUMBER(S) Miscellaneous Paper ITL-90-5		5. MONITORING ORGANIZATION REPORT NUMBER(S)	
6a. NAME OF PERFORMING ORGANIZATION USAWE&S, Information Technology Laboratory	6b. OFFICE SYMBOL (if applicable)	7a. NAME OF MONITORING ORGANIZATION	
6c. ADDRESS (City, State, and ZIP Code) 3909 Halls Ferry Road Vicksburg, MS 39180-6199		7b. ADDRESS (City, State, and ZIP Code)	
8a. NAME OF FUNDING / SPONSORING ORGANIZATION US Army Corps of Engineers	8b. OFFICE SYMBOL (if applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c. ADDRESS (City, State, and ZIP Code) Washington, DC 20314-1000		10. SOURCE OF FUNDING NUMBERS	
		PROGRAM ELEMENT NO.	PROJECT NO.
		TASK NO.	WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification) Review of Finite Element Procedures for Earth Retaining Structures			
12. PERSONAL AUTHOR(S) Ebeling, Robert M.			
13a. TYPE OF REPORT Final report	13b. TIME COVERED FROM _____ TO _____	14. DATE OF REPORT (Year, Month, Day) December 1990	15. PAGE COUNT 45
16. SUPPLEMENTARY NOTATION Available from National Technical Information Service, 5285 Port Royal Road, Springfield, VA 22161.			
17. COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP	
			Base separation
			Finite elements
			Soil-structure interaction
			Basement walls
			Fracture mechanisms
			Earth pressures
			Retaining walls
			U-frame locks
19. ABSTRACT (Continue on reverse if necessary and identify by block number)			
<p>This miscellaneous paper presents a review of previous work in which the finite element method was used to analyze the soil-structure interaction of earth retaining structures such as U-frame locks, gravity walls, and basement walls. This method of analysis results in the computation of stresses and displacements for both the structure and the soil backfill. Applications of the procedure have shown the importance of modeling the actual construction process as closely as possible and the use of a nonlinear stress-strain soil model. Additional requirements include modeling the interface between the soil backfill and the wall using interface elements.</p> <p>This paper also includes two recent applications of the finite element method for the analysis of earth retaining structures which are loaded so heavily that a gap develops along the interface between the base of the structure and its foundation. The results are compared to those computed using the conventional force equilibrium method of analysis.</p>			
20. DISTRIBUTION / AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS		21. ABSTRACT SECURITY CLASSIFICATION Unclassified	
22a. NAME OF RESPONSIBLE INDIVIDUAL		22b. TELEPHONE (Include Area Code)	22c. OFFICE SYMBOL

## **PREFACE**

This miscellaneous paper presents a review of finite element procedures for earth retaining structures. This study is part of the research project entitled "Soil-Structure Interaction Study of Walls" sponsored by the Civil Works Research and Development Directorate, Headquarters, US Army Corps of Engineers (HQUSACE), under the Structural Engineering Research Program. Technical Monitor for the project is Mr. Donald Dressler (HQUSACE).

The work was performed at the US Army Engineer Waterways Experiment Station (WES) by Dr. Robert Ebeling, Scientific and Engineering Applications Center, Computer-Aided Engineering Division (CAED), Information Technology Laboratory (ITL). This miscellaneous paper was prepared by Dr. Robert Ebeling. This study is part of a general investigation on soil-structure interaction of walls under the direction of Mr. Reed Mosher, CAED. All work was accomplished under the general supervision of Dr. Edward Middleton, Chief, CAED, and Dr. N. Radhakrishnan, Chief, ITL.

COL Larry B. Fulton, EN, is Commander and Director of WES. Dr Robert W. Whalin is the Technical Director.

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**CONVERSION FACTORS, NON-SI TO SI (METRIC)  
UNITS OF MEASUREMENT**

Non-SI units of measurement used in this report can be converted to SI (metric) units as follows:

<u>Multiply</u>	<u>By</u>	<u>To Obtain</u>
feet	0.3048	metres
inches	2.54	centimetres
miles (US statute)	1.609347	kilometres

# REVIEW OF FINITE ELEMENT PROCEDURES FOR EARTH RETAINING STRUCTURES

## PART I: INTRODUCTION

1. The purpose of this paper is to present a review of previous work in which the finite element method was used to analyze the soil-structure interaction of earth retaining structures. The finite element method of analysis has been applied to a variety of earth retaining structures and used to calculate stresses and movements for problems involving a wide variety of boundary and loading conditions. Some of the modeling features to be considered in a successful soil-structure interaction analysis are summarized in this paper, along with the results from select soil-structure interaction analyses.

2. Experience with the application of the finite element method in the analysis of stresses and displacements of earth masses has shown the importance of modeling the actual construction process as closely as possible and the inclusion of a nonlinear stress-strain soil model. Application of this procedure to soil-structure interaction analyses has led to the additional requirements that the soil backfill and interface elements be incorporated within the finite element mesh. The first section of this report describes the procedures used in the calculation of earth pressures and displacements using the finite element method of analysis.

3. In recent studies analytical models using the finite element method of analysis have been applied to earth retaining structures which are loaded so heavily that a gap develops along the interface between the base of a structure and its foundation. Two analytical procedures used to model the loss of contact between a structure and its foundation are summarized in the second half of this paper.

## **PART II: CALCULATION OF EARTH PRESSURES AND DISPLACEMENTS USING THE FINITE ELEMENT METHOD**

4. Procedures for the finite element analysis of conventional, stable earth retaining structures are well established. They have been successfully applied to the evaluation of the soil-structure interaction for a variety of earth retaining structures during the past 20 years, including U-frame locks, gravity walls, and basement walls. This section summarizes the key aspects of these types of analyses.

### **Study by Clough and Duncan (1969)**

5. One of the earliest studies was performed by Clough and Duncan (1969) in their analysis of the two reinforced concrete U-frame locks at Port Allen and Old River that had been extensively instrumented. A cross section of Port Allen lock is shown in Figure 1a. In these studies, the soil backfill was represented in the finite element mesh as shown in Figure 1b. During preliminary analyses, it was found that a gravity turn-on analysis was insufficient for the analysis of soil-structure interaction problems. The authors recognized that the analytical procedure used must take into account the nonlinear stress-strain response of soils during loading. In addition, it was shown in these studies that the best agreement is obtained when the actual construction process was simulated as closely as possible. During the course of this study, the authors developed what is referred to as a backfill placement analysis where the loads exerted by the backfill on the lock wall were generated automatically during simulated placement of backfill behind the wall. This procedure involved the use of incremental finite element analysis with nonlinear, stress-dependent, stress-strain behavior for the soil. Linear elastic behavior was assumed for the concrete lock wall.

6. An additional analytical feature used in the Port Allen and Old River study was the inclusion of the Goodman, Taylor, and Brekke (1968) interface elements between the concrete lock walls and the soil backfill. In a traditional finite element analysis using conventional elements, the interface between the backfill and the wall is constrained so that both move in the same direction and are of equal magnitude. In actuality, there is no such constraint on the backfill and wall. This constraint influences both the resulting displacements and computed stresses within the wall and the backfill. The presence of interface elements between the backfill and the wall allows the backfill to move independent of the wall.

7. Clough and Duncan found that their developed procedures gave results in good agreement with the results of the extensive instrumentation program for Port Allen lock and Old River lock. Examples of the agreement between computed and measured displacements and earth pressures for Port Allen lock are shown in Figures 2 and 3, respectively. Seasonal changes were also able to be accounted for in the analyses, as shown in Figure 4. During the winter the lock walls moved away from the backfill, while during the summer the walls moved to displace the backfill. The changes in both the measured and the computed earth pressures were in agreement and consistent with the displacement of the wall. In addition, these changes explained a curious aspect in the

behavior of Port Allen lock; when the lock was filled with water, the walls moved inward, yet the soil pressures acting on the walls increased rather than decreased as intuition would suggest. The results from the finite element analyses showed that the increase in earth pressures was a result of the mass flow of the soil around the lock, as shown in Figure 5.

### **Study by Clough and Duncan (1971)**

8. In a 1971 study, Clough and Duncan showed that nonlinear, incremental finite element procedures could be used to predict lateral earth pressures for conditions ranging from an unmoving wall to limit conditions where the wall displaced enough to generate active or passive earth pressures. A 10-ft\*-high wall retaining a sand backfill and founded on rock (Figure 6a) was used in this analysis. The corresponding finite element mesh is shown in Figure 6b. Interface elements were placed between the wall-to-soil interface and between the rock-to-soil interface. The computed relationships between wall movements and the resultant horizontal earth pressure force, shown in Figure 7, were found to be in good agreement with classical earth pressure theories and the computed deformations were in agreement with those measured by Terzaghi (1934) in his retaining wall tests. In both this study and the study by Nakai (1985), the use of interface elements along the soil-to-wall interface, with varying levels of wall roughness, was shown to influence the computed earth pressures.

9. In the 1971 paper by Clough and Duncan, the authors describe a backfill placement analysis simulating the construction of a 20-ft-high earth retaining wall founded on sand. The sequential construction and backfilling simulation, idealized in Figure 10, was performed using the incremental, nonlinear finite element method of analysis described in paragraph 3. A total of eight construction increments were used; the wall was completed by the end of the second increment and backfilling was completed by the end of the eighth increment. Interface elements were used along the wall-to-soil interfaces and along the base of the wall. The calculated deflections shown in exaggerated scale in Figure 11 show that the wall tilted towards the backfill during construction, rather than away from the backfill as classical earth pressure theories (e.g., the theory for active earth pressures), would indicate. Careful examination of this figure reveals that the wall moved and tilted forward relative to the backfill. This resulted in earth pressure forces from the finite element analysis greater than those computed using the classical earth pressure theory for an active stress state (Figure 12), but less than at-rest values. Two contributing factors are the incorporation of the compressibility of the foundation in the analysis and the non-uniform loading of the foundation sands. Lastly, the results showed that a stabilizing shear force acting along the back of the wall (referred to as a downdrag force) could develop during backfill placement simply due to compression of the backfill soil under its own weight. This finding was important in that prior to this it was believed that a downdrag force occurred only as a

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\* A table of factors for converting non-SI units of measurement to SI (metric) units is presented on page 3.

result of the movement of the wall away from the backfill in response to the earth loadings. In addition, the inclusion of interface elements along the material interface regions within the mesh allowed the soil to settle during backfilling while the wall moved away from the backfill.

### **Study by Bhatia and Bakeer (1989)**

10. Bhatia and Bakeer (1989) performed a finite element analysis of a 10-m-high instrumented experiment wall resting on a hinged base, shown in Figure 8a, that was tested by Matsuo, Kenmochi, and Yagi (1978). Their basic finite element mesh is shown in Figure 8b and contains interface elements between the backfill and the wall. A series of analyses similar to the Clough and Duncan analyses described in paragraph 6 were conducted for the boundary conditions ranging from a wall with zero rotation to the case where the crest of the wall was rotated 0.016 m. There is reasonable agreement between the measured and predicted earth pressures, as shown in Figure 9, particularly for the rotated wall case.

### **Study by Kulhawy (1974)**

11. Kulhawy (1974) performed analyses of a proposed 104-ft-high gravity earth retaining wall shown in Figure 13a. The wall, which was to be founded on rock, was analyzed using the Duncan and Clough backfill placement analysis procedure. Their finite element mesh (shown in Figure 13b) included both soil-to-concrete interface elements behind the wall and concrete-to-rock interface elements between the wall and the rock foundation. Due to the mass of the wall, the computed wall movements were very small, less than 1 in. of lateral movement at the crest. Thus, the resulting earth pressures after the completion of backfilling were nearer to their at-rest pressures than their active values, as shown in Figures 14a and 14b. Downdrag forces were computed along the back of the wall, as was the case for the Duncan and Clough retaining wall analyses described in paragraph 10. Another observation was that the resulting earth pressures and deformation of the wall were dependent upon the material parameters assigned to the backfill, i.e., the value of the stiffness and the value of Poisson's ratio.

### **Study by Roth, Lee, and Crandall (1979)**

12. Roth, Lee, and Crandall (1979) described backfill placement analysis of an instrumented, deep basement wall, using the same finite element procedure as Clough and Duncan (1969). The finite element mesh used in the analysis is shown in Figure 15a. The variation in readings for two of the pressure cells during the backfilling of the wall are shown in Figures 15b and 15c, along with the finite element results. The instrumentation measurements after completion of backfilling are compared to the computed results in Figure 15d. Good agreement was found between calculated and measured lateral earth pressures when interface elements were included along the backfill-to-wall interface. By using interface elements in the finite element analyses of a rigid wall, they were able to simulate the settlement of the backfill adjacent to the wall, resulting

in the mobilization of a shear force along the back of the wall. In the parametric analyses, they found that the value of Poisson's ratio assigned to the backfill was the most important parameter affecting the calculated lateral earth pressure, and the stiffness assigned to the backfill had little influence on the calculated lateral pressures.

### Studies by Ebeling et al. (1988) and Ebeling, Duncan, and Clough (1989)

13. Ebeling et al. (1988) and Ebeling, Duncan, and Clough (1989) describe a series of backfill placement analyses of gravity earth retaining structures of the type shown in Figure 16a. The finite element mesh for this wall is shown in Figure 16b. It was shown during the course of this study that there is an interdependence between wall deformations and the distribution of both stabilizing and destabilizing forces exerted on the wall by the fill, and on the base of the wall. The relative distribution of these forces was found to be dependent upon the size of the wall, its proportion of base to height, the geometry of the wall, and whether the face along the back of the wall was stepped or planar. The presence of water also influences the distribution of forces acting on the wall. In addition, the material properties of the backfill, the rock foundation, and the soil-to-concrete and concrete-to-rock interface regions influenced the computed results.

14. It was observed that for walls of typical geometry, unless there were special regions with unique material properties that would contribute to significant wall deformations, the magnitude of the wall movements away from the backfill is very small. For example, less than 1 in. of lateral movement was computed after completion of backfilling for the 40-ft-high by 16-ft-wide wall shown in Figure 16. The resulting lateral earth pressures for the backfill were closer to their at-rest values than their active values. The analyses demonstrated that the backfill settles more than the wall, and develops a downward acting shear stress on the back of the wall, as shown in Figure 17. The shear stresses were expressed in terms of a resultant shear force,  $F_v$ , acting along the vertical plane. When the backfill is dry,  $F_v$  may in turn be conveniently expressed in terms of the vertical earth pressure coefficient,  $K_v$ , by the following equation:

$$F_v = K_v \cdot \frac{1}{2} \gamma H^2 \quad (1)$$

where

$H$  = height of backfill

$\gamma$  = unit weight of backfill

Figure 17 shows that  $F_v$  decreases to a near zero value at a distance equal to 40 feet, as measured from the heel of the wall, a distance equal to the height of the wall. This shear force is a stabilizing force acting on the back of the wall, tending to counter the lateral earth pressure forces attempting to destabilize the wall. Figure 18 shows the variation in  $K_v$  for walls 40 ft in height and a base width equal to 16 ft but with different wall geometry.  $K_h$  is the lateral earth pressure coefficient, the ratio ( $B_e/B$ ) the

effective base area remaining in compression,  $q_{\max}$  the maximum compressive stress developed along the base and  $\delta_{\text{mob}}$  is the mobilized angle of friction along the base of the wall. The presence of this downdrag force provides an explanation for the anomaly that although existing Corps gravity earth retaining walls at various US navigation lock sites have been judged unstable on the basis of current Corps design methods, they are in fact performing well, without signs of instability.

### **Results of Finite Element Studies**

15. The previously described finite element studies of U-frame locks, basement walls, and retaining walls led to a number of common conclusions:

- a. Modeling procedures yield results closest to observed behavior when the wall and backfill construction sequences are simulated.
- b. The model should include provisions to account for the nonlinear stress-strain behavior exhibited by soils. Incremental, equivalent linear techniques have proven to be quite successful for a variety of projects.
- c. Representation of the interface between the structure and the soil is important to obtain realistic results. The relative movement along the interface must be included in the finite element analysis.

16. These finite element analyses shed light on the soil-to-structure interaction in a way not possible otherwise. As such, they were very useful. Some of the more important findings attributed to the finite element analyses of the earth retaining structures are:

- a. The finite element analysis can capture the interdependence between wall deformations and the distribution of both stabilizing and destabilizing forces exerted on the base of the wall and on the wall by the backfill.
- b. Finite element analysis of wall-backfill systems can represent conditions from at-rest to the limit states.
- c. Downdrag can develop on a wall due to downward movements of the soil backfill relative to the wall as a result of only the compression of the backfill solid under its own weight. Classical earth pressure theory assuming limiting states of stress within the backfill, such as an active stress state, would lead one to believe that the downdrag force would only develop when the wall moved away from the backfill.

17. An additional complication in the soil-to-structure interaction analysis of walls using the finite element method of analysis is the situation where the earth and/or water loads acting on the structures are so great that a gap develops along the interface between the base of a wall and its foundation. When interface elements are used to model the interface between the wall and its foundation, the inability to prevent the interface elements from assuming stresses higher than the allowable values becomes a problem. This is termed "overshoot." Although the overshoot error in any one element may be small, the error can accumulate, potentially leading to unsatisfactory results. Numerical procedures have recently been developed to resolve this problem and will be reviewed in the next section.

### **PART III: LOSS OF CONTACT BETWEEN STRUCTURE AND FOUNDATION**

18. In recent years research efforts have been directed towards the development of analytical procedures, using the finite element method of analysis, to analyze problems concerned with loss of contact between the base of a wall and its foundation. This situation arises when structures are loaded so heavily that a gap develops within the interface region. Two analytical approaches have been used to analyze this type of problem; one procedure involves the modelling of a predetermined plane along which separation is presumed to develop using interface elements and the second analytical procedure involves the use of concepts associated with fracture mechanics. Both procedures involve the use of the finite element method of analysis to determine the response of a structure to earth and water loadings, described in terms of the change in displacements and stresses, and both procedures have been applied to soil-structure interaction analyses of earth retaining structures.

#### **Base Separation Analyses Using Interface Elements**

19. There have been a limited number of investigations in which the finite element method has been adapted to the problem of a loss of contact between the base of a footing and its foundation through the use of interface elements. Desai, Mistry, and Patel (1985) and Herrmann (1978) describe the development of procedures for modeling the loss of contact between a strip footing and the soil foundation, when the footing is subjected to an eccentric vertical load. In both analyses, the soil was modeled as an elastic continuum. In the study by Ebeling et al. (1988) a similar model was developed and used in the analysis of earth retaining structures.

20. In the problem studied by Desai, Mistry, and Patel (1985) a strip footing, resting on the surface of an overconsolidated clay, was subjected to a single eccentric vertical load. The finite element mesh for this problem is shown in Figure 19a. Interface elements were included between the footing and the soil. The formulation of the interface element was based on the same constitutive relationships as those used by Goodman, Taylor, and Brekke (1968). The procedure of analysis was one of successive iterations for each eccentric vertical load applied to the footing. An elastic analysis was initially performed with the assumption that the soil-to-footing interface can transmit full tension. The resulting pressure distribution within the interface elements for this analysis is shown in Figure 19b and labelled "full contact." Each interface element was checked for the development of tensile stress. When tensile stresses were observed, the interface stiffnesses were set equal to zero. The problem was re-analyzed for the same eccentric load using the updated stiffnesses. When all active interface elements were found to be in compression, an elastic solution to the problem was reached. The resulting distribution of normal stresses is shown in Figure 19b and labelled loss of contact. In the problem analyzed by Desai, Mistry, and Patel (1985) it was observed that the ability to model the loss of contact resulted in: (a) an increased maximum contact compressive stress, (b) an increase in the rotation of the footing, and (c) an increase in the

bending moments developed within the footing, as compared to the results from the conventional elastic continuum and spring bed models for soil behavior. It is important to note that the problem for which this procedure was developed does not include shear stresses along the interface which are present in problems involving retaining walls.

21. The problem studied by Herrmann (1978) was similar to that described previously of a rigid footing resting on an elastic foundation with interface elements between the two regions. The algorithm used for this interface element was very similar to that for interface elements developed by Goodman, Taylor, and Brekke (1968), but with additional constraint conditions introduced. This bond-link interface element uses equivalent shear and normal springs at the nodes and allows slip and/or separation to develop. Like Desai's, the procedure was one of successive iterations for each eccentric vertical load applied to the footing, checking for compatibility and equilibrium along the interface region. The results for the analysis of the problem of a rigid footing on an elastic foundation indicated that for a given value of load, the rotation of the footing increased in proportion to the eccentricity of the load, as shown in Figure 20. However, once uplift occurred, the rotation of the footing increased in a nonlinear manner with increased eccentricity in the applied load.

22. The results from the two analyses of eccentrically loaded footings indicate that consideration of the loss of contact along the base of a structure affects the magnitude of the rotation, and the magnitude and distribution of the stresses developed along the base of the structure. These studies indicate that in a soil-to-structure interaction evaluation of an earth retaining structure loaded so heavily that a gap develops along the base, the ability to model the loss of contact along the base would be an important feature. This is due to the fact that base separation influences both the magnitude of the computed wall rotation and, therefore, the resulting earth pressures acting on the wall.

23. Due to the interrelationship between wall movements and the distribution of both stabilizing and destabilizing forces exerted on the wall by the fill, a procedure for modelling the loss of contact was developed by Ebeling et al. (1988). The procedure, referred to as the Alpha method, was implemented within the framework of the incremental, equivalent linear backfill placement analysis procedure discussed in paragraph 3. During the course of the incremental analysis, each interface element along the base of the wall is checked for the development of tensile stress at its center. If none are found, the backfill placement analysis proceeds as usual. When tensile stresses are observed in the interface elements, the incremental analysis is repeated with the Alpha method implemented. Briefly, the principle of the procedure is to: (a) factor the applied incremental load vector so that zero normal stress will result at the center of each of the interface elements which previously developed tensile stress at its center, (b) make the interface stiffnesses equal to zero, (c) convert the shear stress regime into an equivalent set of nodal point forces, (d) transfer this equivalent force into adjacent elements by applying it as an external force at the nodes, and (e) maintain equilibrium by subtracting the equivalent internal stress from within the interface element(s) used to formulate this force. The procedure is repeated until the total initial load increment has

been applied. The name given to this method is derived from the factor applied to the incremental load vector, "Alpha."

24. The accuracy of the Alpha method is best shown by comparing the results using an incremental method of analysis to the results from the Alpha method. The wall analyzed, shown in Figure 21a, is 40 ft high and 16 ft wide at the base and founded on competent rock. The wall was loaded by three basic force components. The first involves the vertical loads induced by the weight of the monolith and the weight of backfill above the heel of the wall. The second component is the lateral stress assumed to be generated by the soil backfill and the hydrostatic water in the backfill ( $H_w = 27$  ft). The third loading is the upward pressure acting on the base of the wall generated by hydrostatic uplift. The gravity loading is applied first, followed by the application of lateral and uplift pressures in 10 load increments shown in Figure 21a. The finite element mesh for the structure is shown in Figure 21b. In both finite element analyses, the gravity loads are applied first, followed by the 10 increments of loading. When tensile stresses are sensed in an interface element during any incremental finite element analysis, the normal and shear stiffnesses are set equal to zero in the interface element and the incremental analysis proceeds with the next load increment. Base separation is assumed to have occurred within this interface element. However, since the loading is discrete, the stress within that interface element may not be exactly zero, which is the usual case.

25. The resulting normal and shear stress distribution along the base of the wall, upon completion of the earth pressure loading (load case 5 in Figure 21a), is shown in Figure 22. Both procedures indicate that a gap has developed along the base of the wall by this stage of loading but its magnitude differs. The incremental analysis predicts 9 ft of the base remaining in compression,  $B_e$ , while the Alpha method predicts  $B_e$  equal to 5 ft. In addition, using the conventional force equilibrium method of analysis and *assuming* a linear compressive stress distribution,  $B_e$  is computed to be 4.5 ft. The difference in the finite element results is attributed to the numerical inaccuracies introduced during the course of the analysis by the particular base separation model used. These numerical inaccuracies are introduced as the criteria for deciding when each interface element has simulated the development of a gap is implemented.

26. When a gap develops during the course of loading of a wall, no forces are transferred along the separated region of the base. Since the loadings are of finite magnitude in an incremental finite element analysis, the developed stresses within the gap are never exactly equal to zero, as discussed in paragraph 22. Thus, the accuracy of a base separation model may be assessed by converting the residual stress distribution along the gap into an equivalent force, both normal and shear, and comparing its magnitude to the total forces acting on the base. These equivalent forces are obtained by integrating the normal and shear stress distributions within the interface elements which have separated. An exact base separation model would have zero net normal force,  $\Delta N$ , and shear force,  $\Delta T$ , retained within the interface elements comprising the gap. These overshoot forces, normalized by the total normal and shear forces acting on the base, are shown in Figure 23 for each increment of loading. This figure shows that the Alpha

method has near zero overshoot at all stages of loading while the error in an incremental analysis is significant.

27. A second measure of error in a base separation analysis reflects the influence which the overshoot normal force has on the distribution of normal force for those interface elements remaining in compression. In Figure 24a, the location of the resultant normal force for the region in compression as computed by the finite element analyses is compared to that of the conventional equilibrium analysis. If the locations are in agreement, the results would plot on the diagonal line through the figure. It is observed that as the loading increases, and the location of the resultant normal force moves towards the toe, the error in the computed point of action in the incremental finite element analysis increases. In contrast, the results from the Alpha method agree with the results from the conventional equilibrium analysis.

28. Figure 24b shows the variation in the ratio  $B_e/B$ , the effective base area in compression normalized by the total base area, with load increment for the three analyses. As the level of loading increases, the effective base area remaining in compression decreases. However, due to overshoot forces retained within separated interface elements in the incremental analysis, the computed values of  $B_e$  are too large. Lastly, the  $B_e$  values computed using the Alpha method are virtually the same as those obtained using the conventional force equilibrium method of analysis and *assuming* a linear compressive stress distribution. The value of  $B_e$ , or conversely, the computed length of the gap ( $B - B_e$ ), would directly influence the magnitude of uplift pressures applied along the base during the course of the stability evaluation of the wall when water is present in the backfill.

29. In a complete soil-to-structure interaction analysis, the soil backfill would be represented in the finite element mesh and the loadings on the wall developed through the interaction between the soil and the wall during backfilling, as summarized in paragraph 13. The ability to accurately model the development of a gap during the course of loading would have implications on the stresses developed along the base, the resulting wall displacements, and, therefore, the earth pressures acting on the wall.

### **Base Separation Analyses Using Fracture Mechanics Concepts**

30. A second procedure for modelling the development of a crack at the base of an earth retaining structure in a soil-structure interaction analysis involves the use of concepts associated with fracture mechanics. In general, fracture mechanics relates the stress magnitude and distribution at the crack tip to the nominal stress applied to the structure; to the size, shape and orientation of the crack or discontinuity; and to the material properties. Research on fracture mechanics has progressed to the point that it has been used to evaluate a massive concrete earth retaining structure at a Corps navigation structure.

31. Monolith 7E at Lock No. 27, located on the Mississippi River 5 miles upstream of St. Louis, MO, was analyzed using both the conventional equilibrium method of analysis and fracture mechanics. At this section, the wall is 92 ft high and 45 ft wide at

the base, as shown in Figure 25a. In both analyses, the soil loads acting on the wall were of predetermined magnitude and thus independent of wall movement, as shown in Figure 25a. Hydrostatic water pressures were assumed along the back of the wall and a pool elevation equal to 340 ft in front of the wall. Full uplift pressures were assumed under the separated region of the base and a linear uplift distribution was assumed under the compression area.

32. The conventional force equilibrium method of analysis, with an *assumed* linear compressive stress distribution along the base, resulted in 48 percent of the base in compression. This does not meet the design requirement of 75 percent for new structures of this type. The computed sliding factor of safety and maximum bearing pressures were indicative of a stable structure.

33. A linear elastic fracture mechanics (LEFM) analysis was performed for a crack along the interface between the concrete wall and the rock foundation. The finite element mesh of the wall and rock foundation is shown in Figure 25b. Uplift pressures were assigned along the base as described in paragraph 29. A crack length of 8 ft was computed using the simplified LEFM analysis. This corresponds to 82 percent of the base area remaining in compression. The computed crest displacements were small. Lateral movement of the crest away from the backfill was 0.09 in. These computed results clearly indicate a stable structure.

34. When the lock was dewatered, no signs of distress were detected and the instrumentation indicated movement at the top of the lock was 0.006 in. away from the backfill. The results for this analysis and those described in paragraphs 18 through 29 indicate that the computed responses of retaining structures are quite sensitive to both the model used for computing the development of a crack within a structure and the presence of water within the crack.

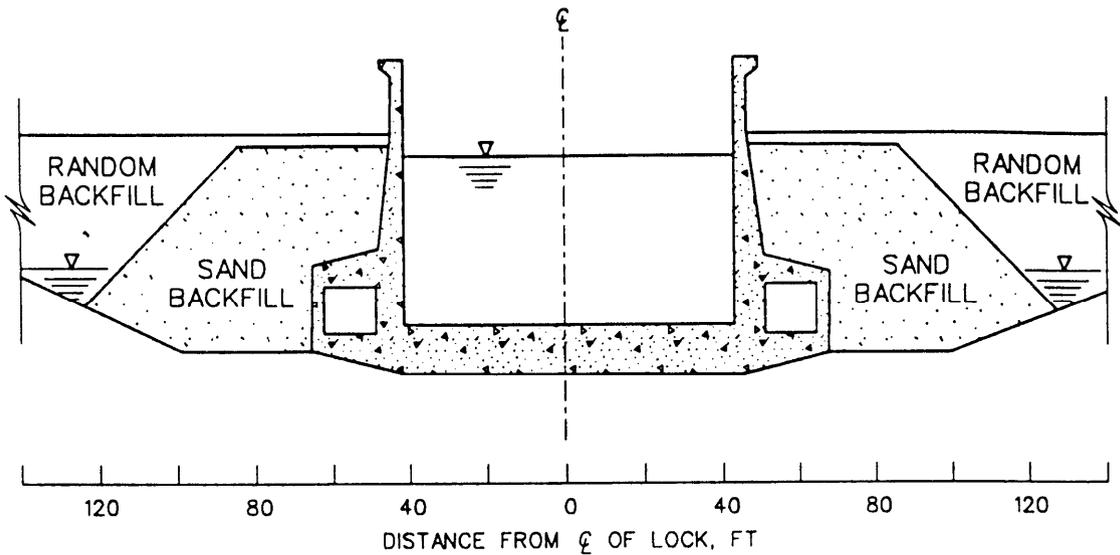
## PART IV: CONCLUSIONS

35. The analyses discussed in this report show the importance of simulating actual construction processes as closely as possible in a finite element analysis. Soil backfill and interface elements should be included in the finite element mesh. In addition, the analysis should account for the nonlinear stress-strain behavior of the soil.

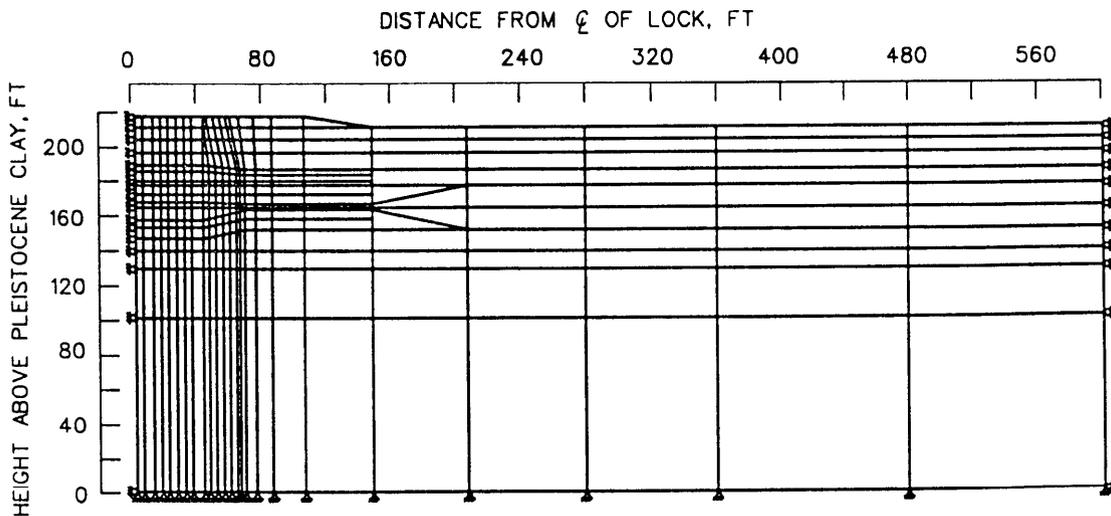
36. Three types of base separation models have been discussed in this report, the first based upon the use of interface elements to model the presumed path of the crack, the second based upon the use of fracture mechanics, and the third based upon the conventional force equilibrium method of analysis with an assumed linear compressive stress distribution. The retaining wall analyses show the importance of using an appropriate base separation model, especially in the presence of water.

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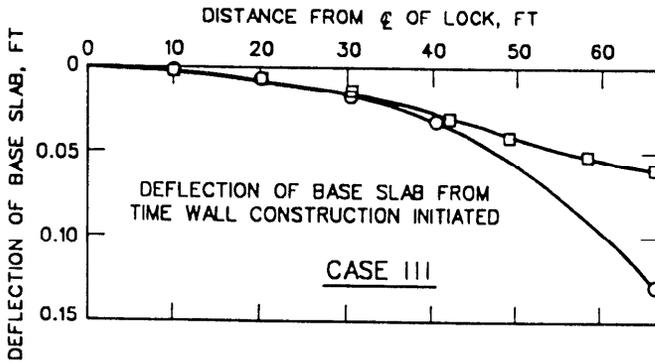
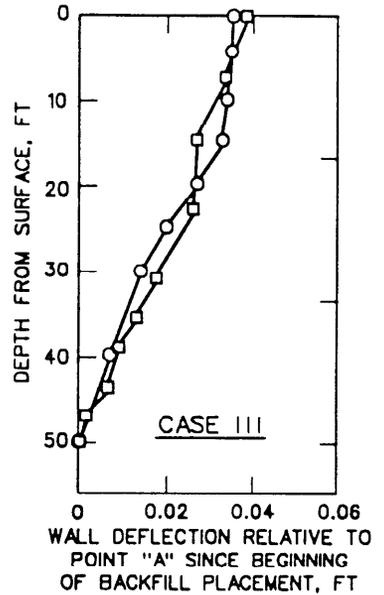
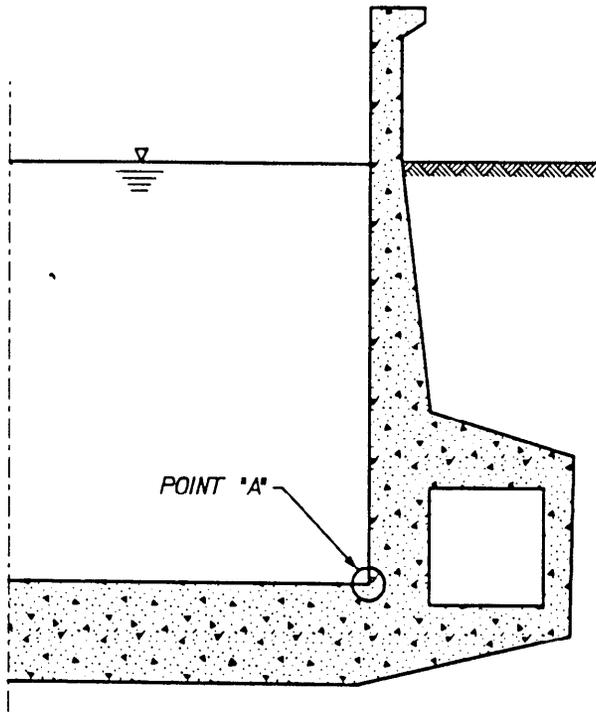


*a. Cross section of Port Allen Lock*



*b. Finite element mesh employed for incremental analyses of Port Allen Lock*

*Figure 1. Cross-section view and finite element mesh for Port Allen Lock (Clough and Duncan 1969)*



LEGEND

○ OBSERVED DEFLECTIONS  
 □ DEFLECTIONS CALCULATED BY F. E. APPROACH

Figure 2. Structural deflections for Case III - Port Allen Lock (Clough and Duncan 1969)

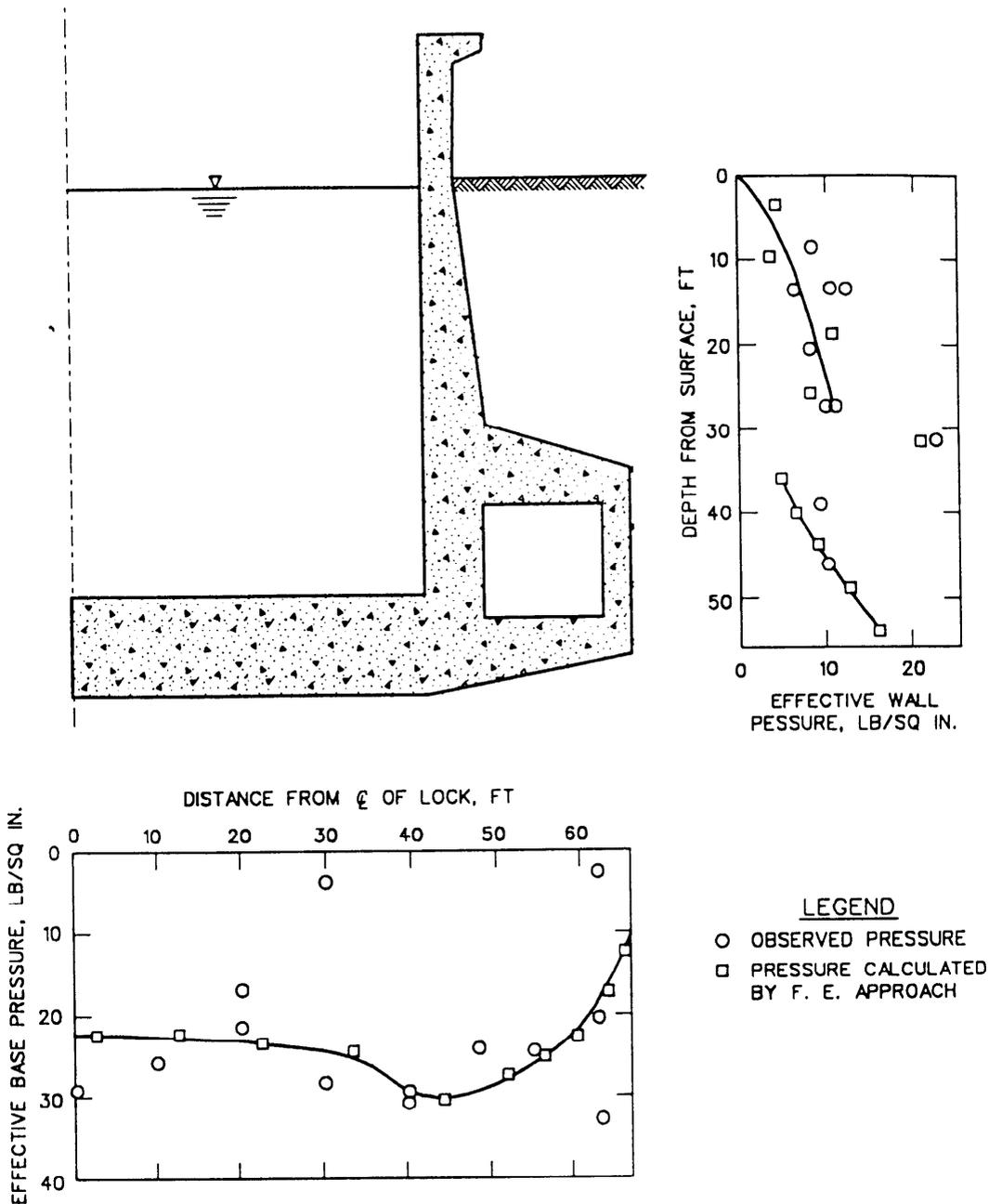


Figure 3. Effective earth pressures for Case III - Port Allen Lock (Clough and Duncan 1969)

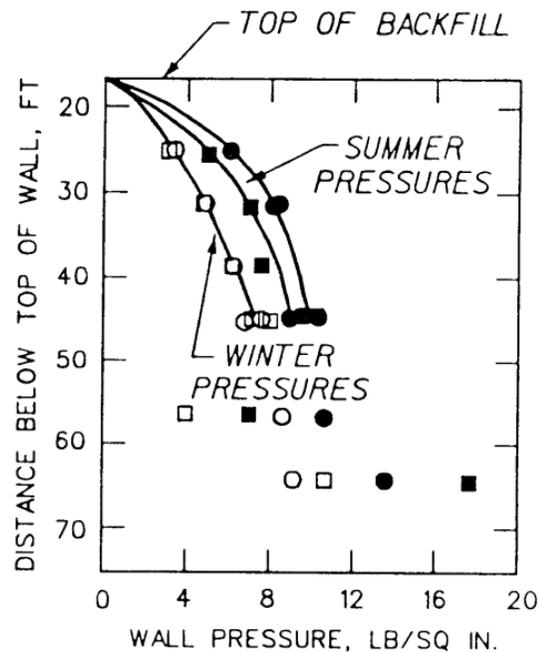
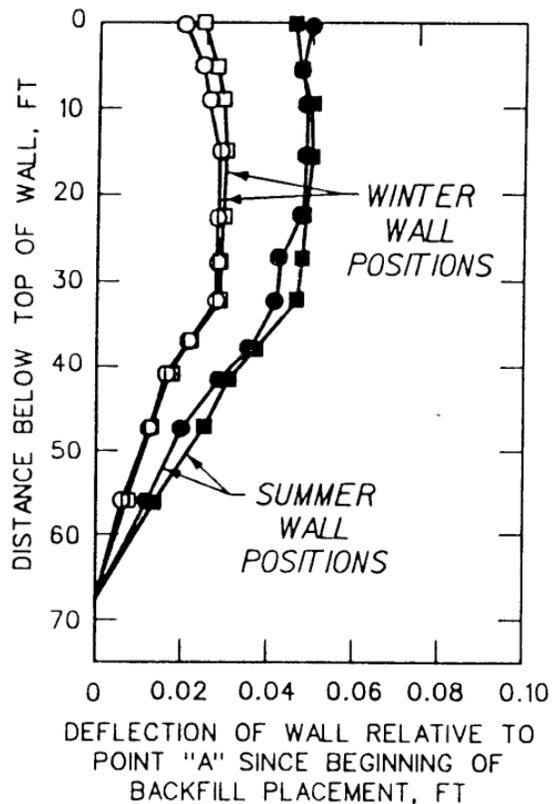
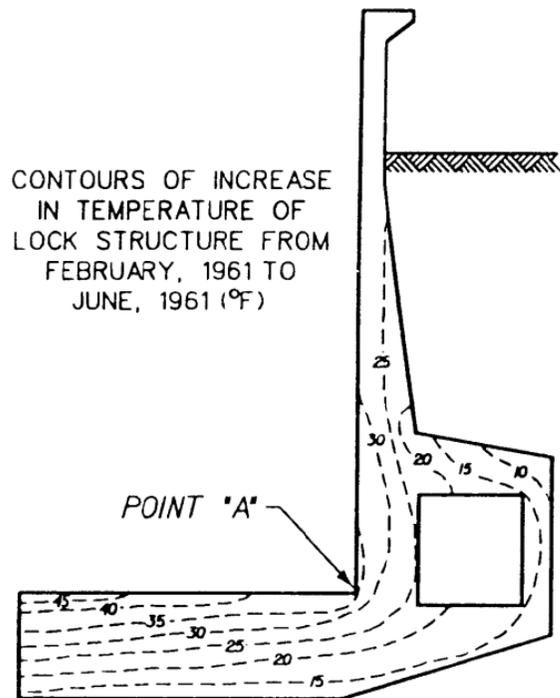
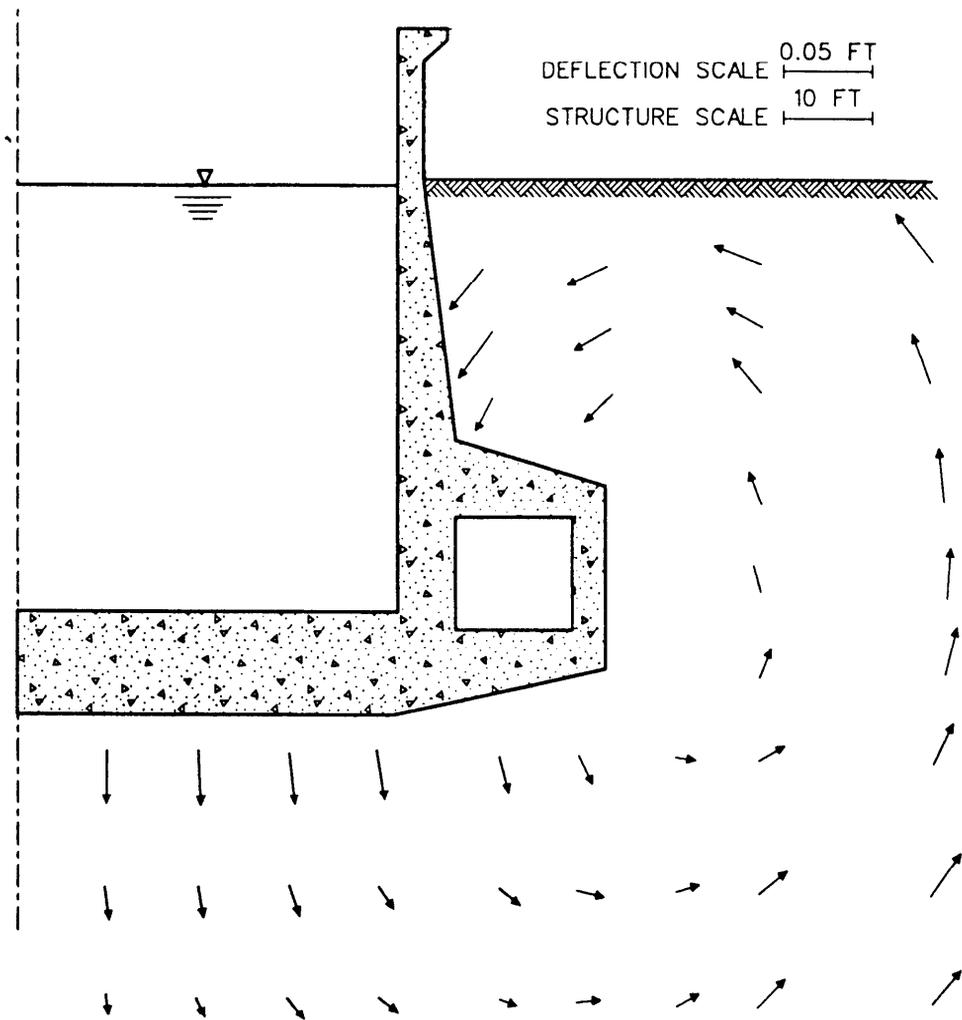
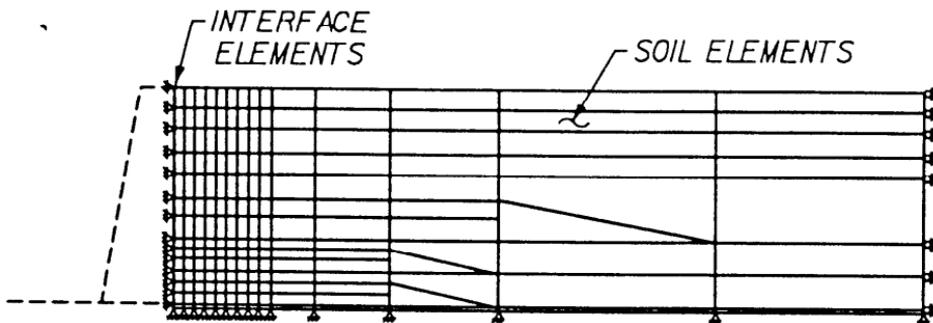


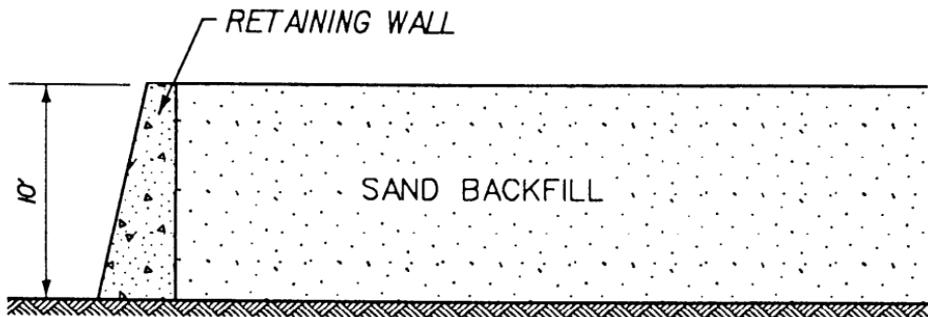
Figure 4. Temperature-induced wall movements and changes in earth pressure at Port Allen Lock (Clough and Duncan 1969)



*Figure 5. Calculated movements due to filling Port Allen Lock with water  
(Clough and Duncan 1969)*

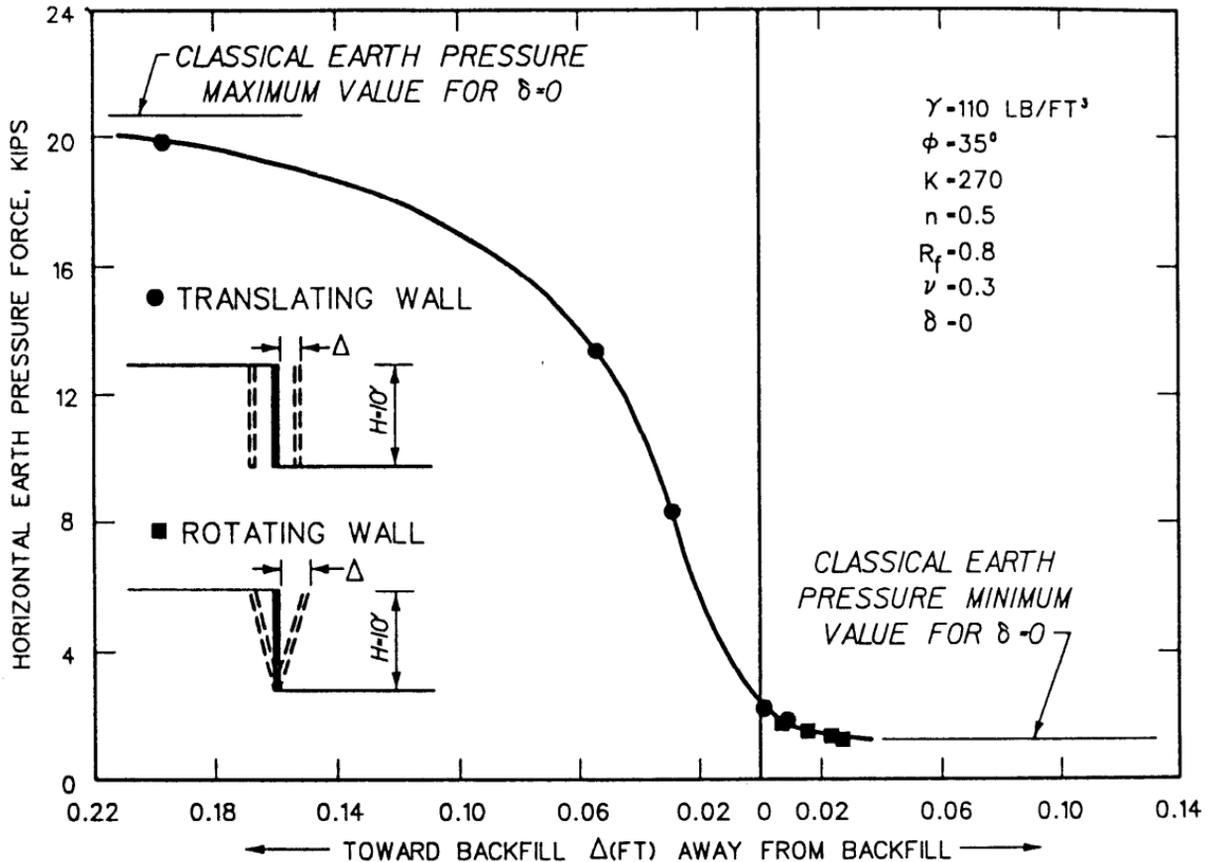


*a. Retaining wall—backfill system*

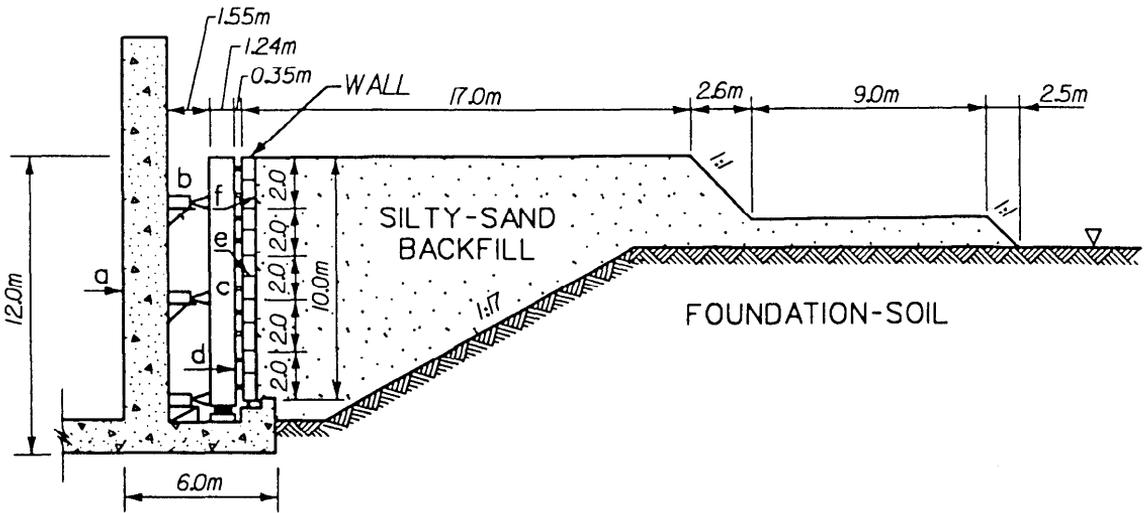


*b. Retaining wall—finite element representation*

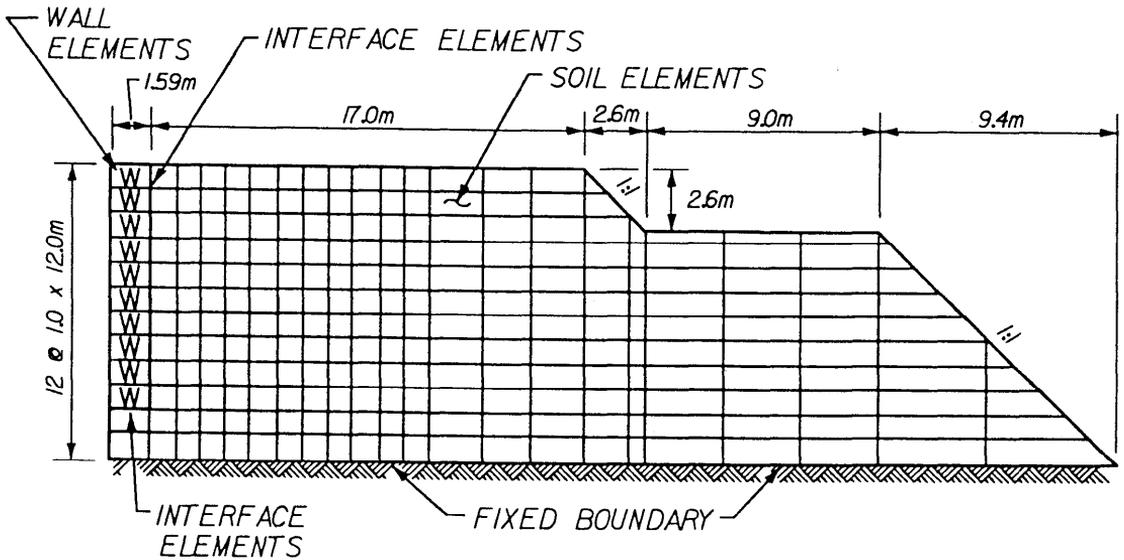
*Figure 6. Cross-section view and finite element mesh for study by Clough and Duncan (1971)*



*Figure 7. Variations of earth pressure force with wall movement calculated by finite element analyses (Clough and Duncan 1971)*



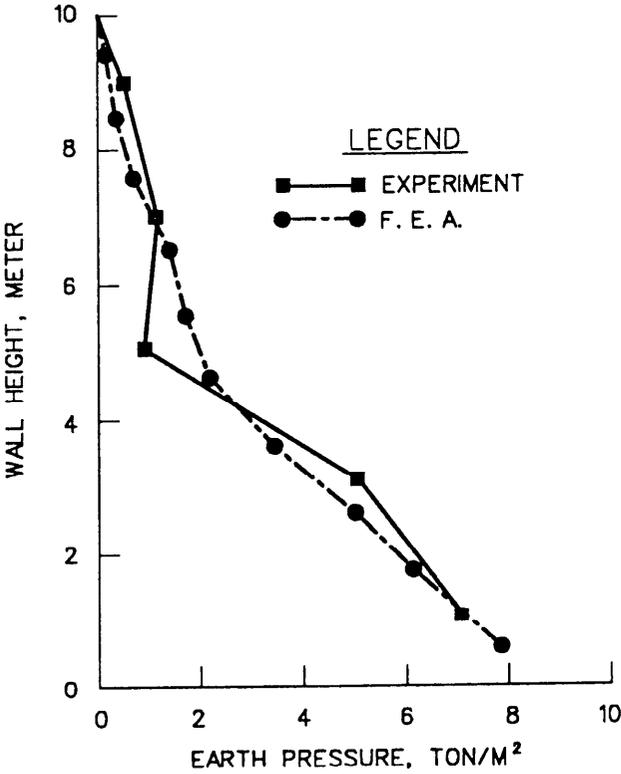
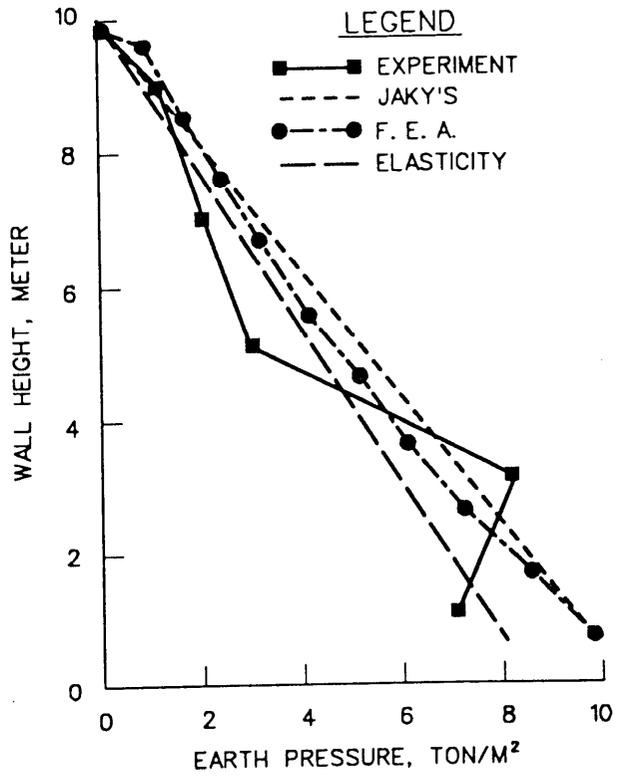
a. Experimental wall model



b. Basic finite element mesh

Figure 8. Cross-section view and finite element mesh for study by S.K. Bhatia and R. M. Bakeer (1989)

a. No displacement



b. Wall rotation of 0.0016 H

Figure 9. Earth pressure distribution in study by Bhatia and Bakeer (1989)

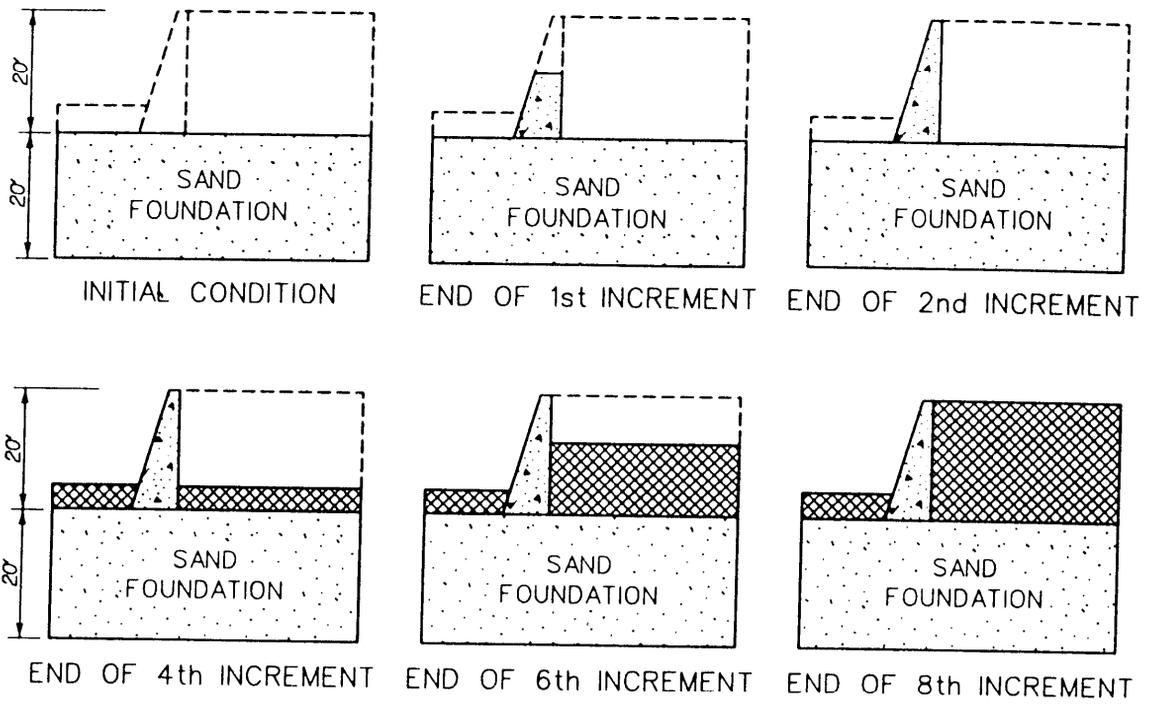


Figure 10. Gravity wall construction sequence (Clough and Duncan 1971)

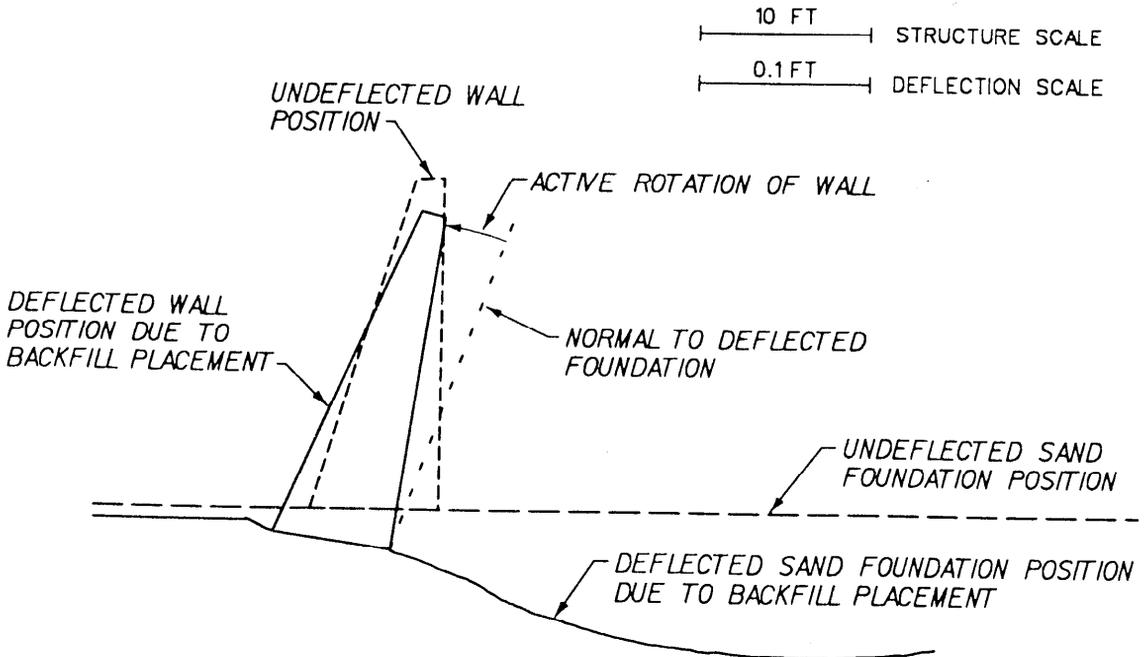


Figure 11. Wall and sand foundation deflections during backfill placement

CLASSICAL EARTH PRESSURE THEORY  
ACTIVE BACKFILL PRESSURES

FINITE ELEMENT APPROACH

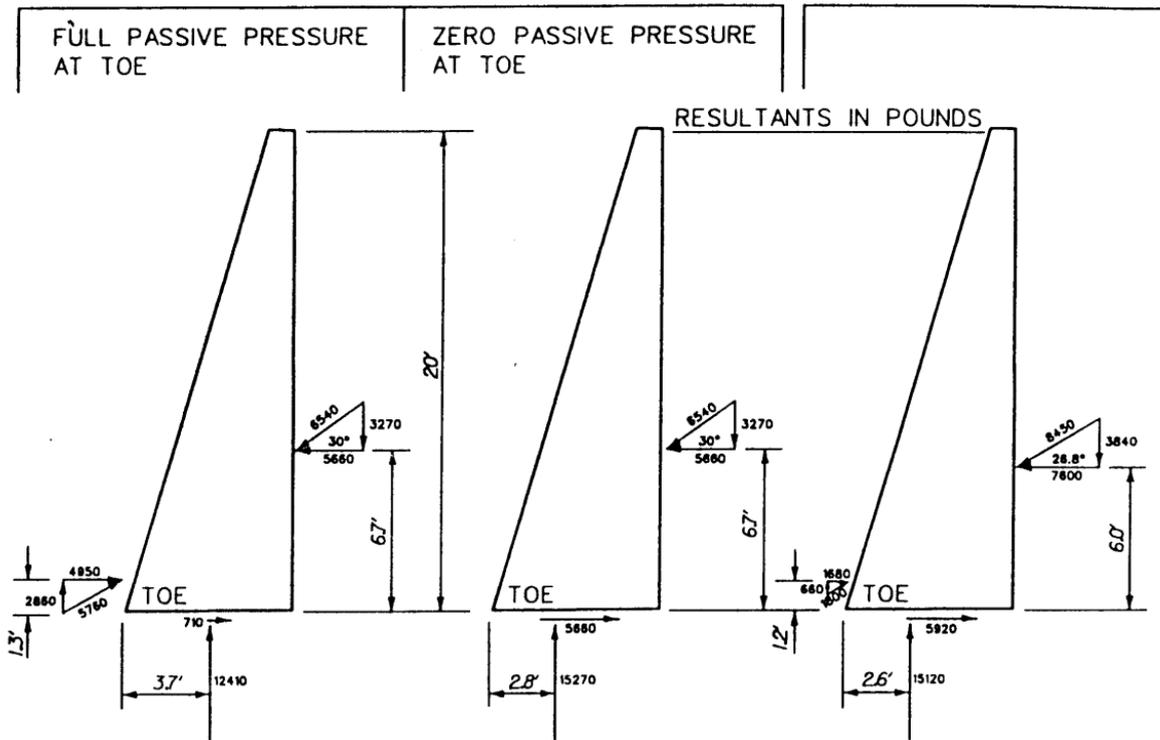
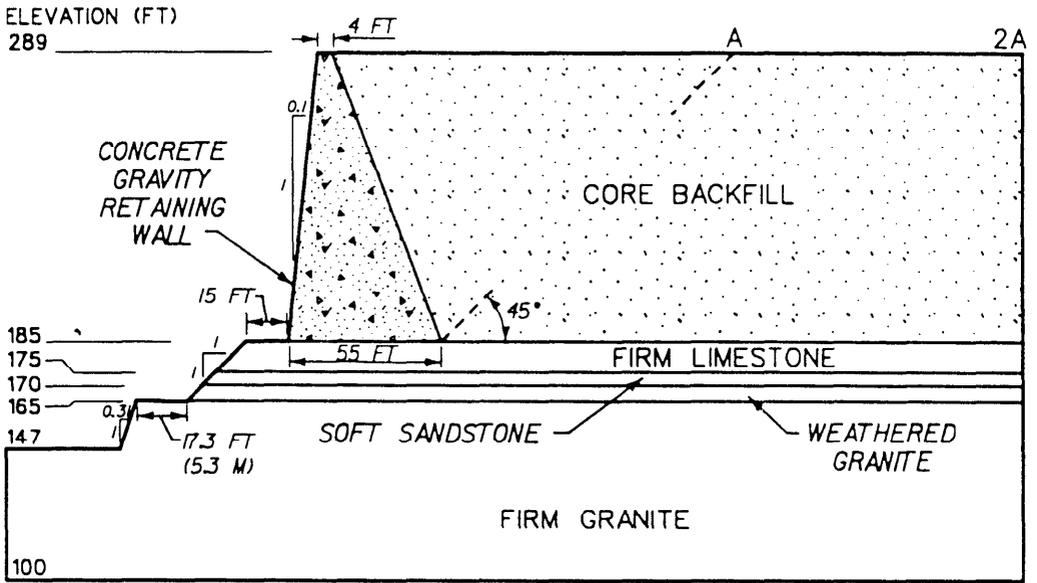
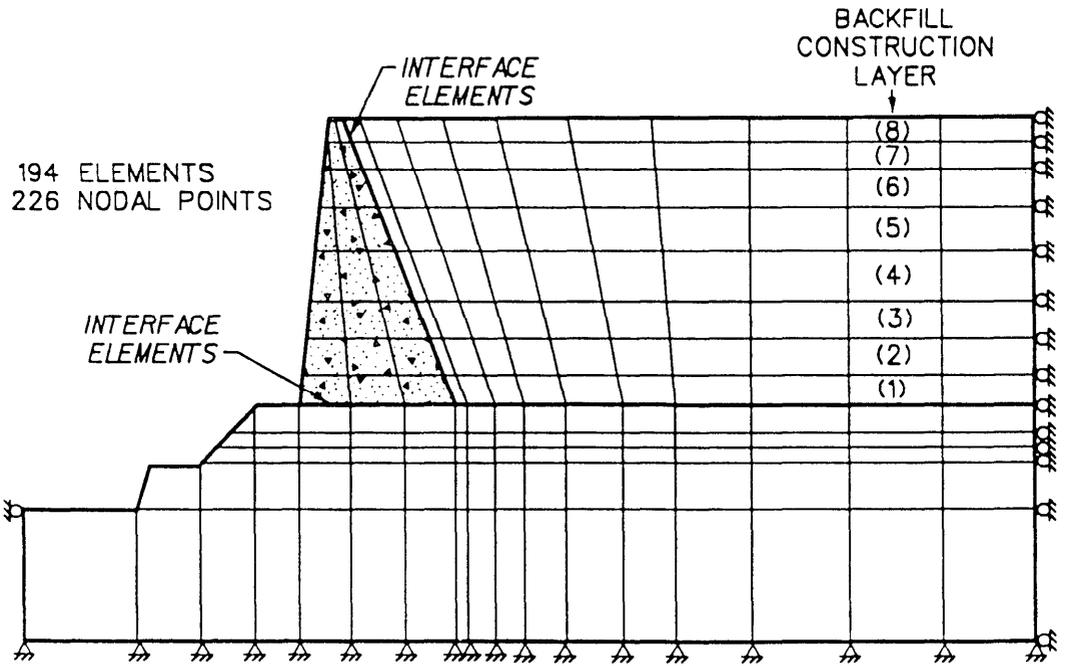


Figure 12. Earth pressure resultants acting on wall from classical earth pressure theory and finite element results (Clough and Duncan 1971)

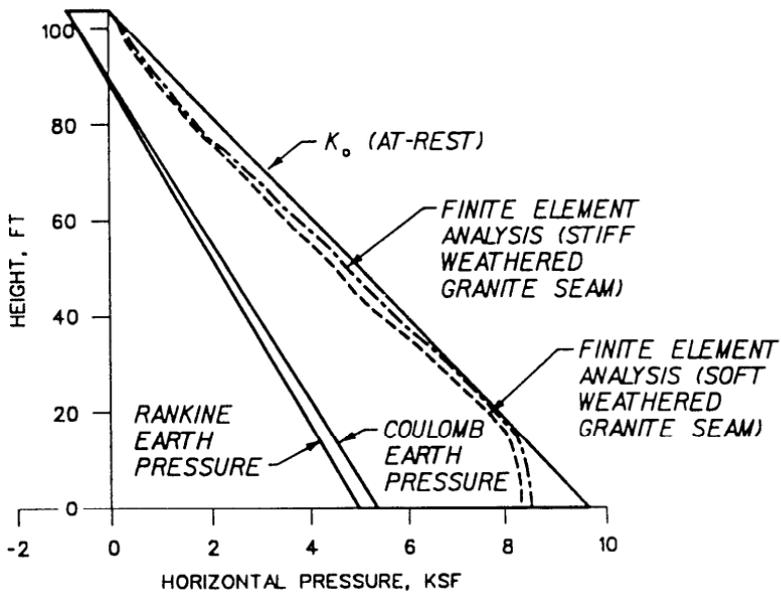


a. Wall modeled section

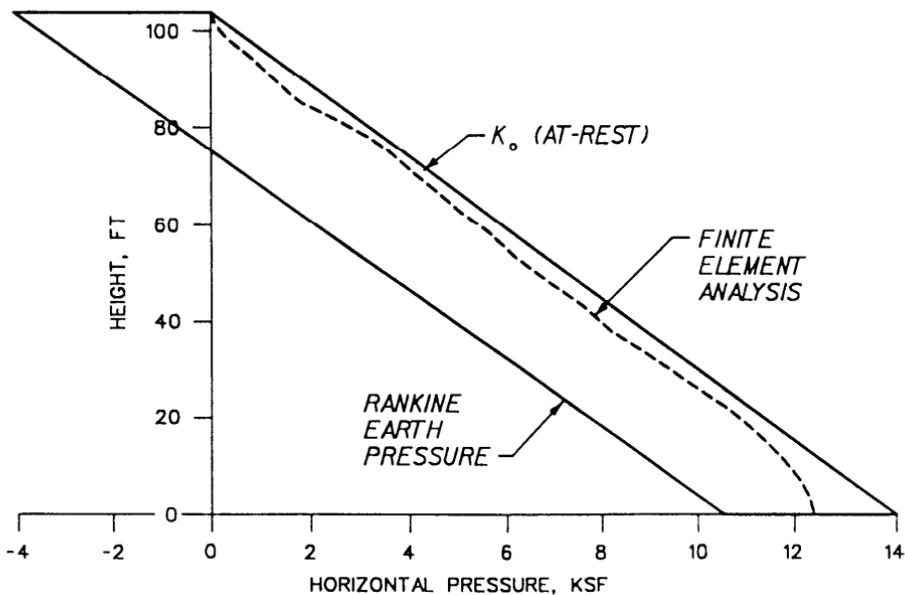


b. Finite element mesh for wall

Figure 13. Wall modeled section and finite element mesh for study by Kulhawy (1974)

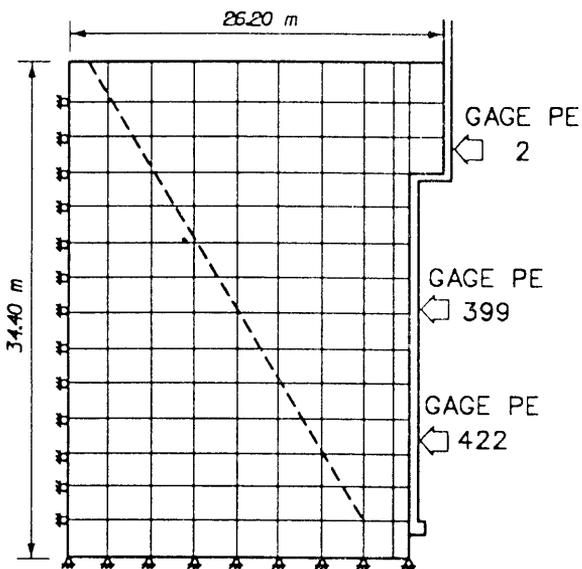


a. Stiff core analysis

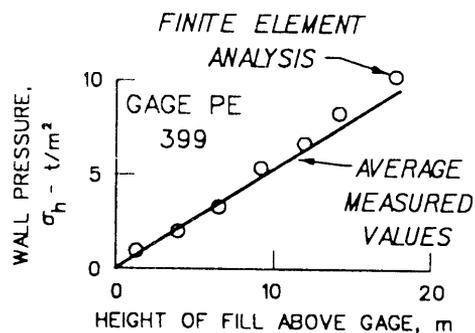


b. Soft core analysis

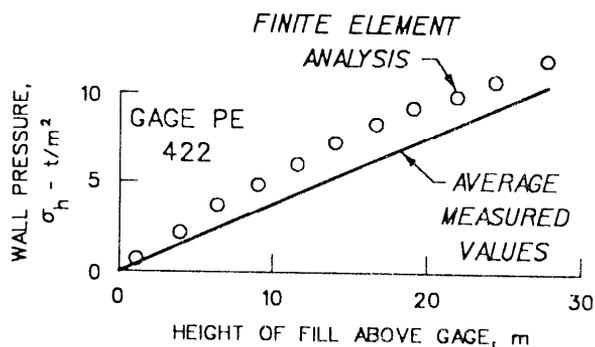
Figure 14. Computed horizontal earth pressures for wall



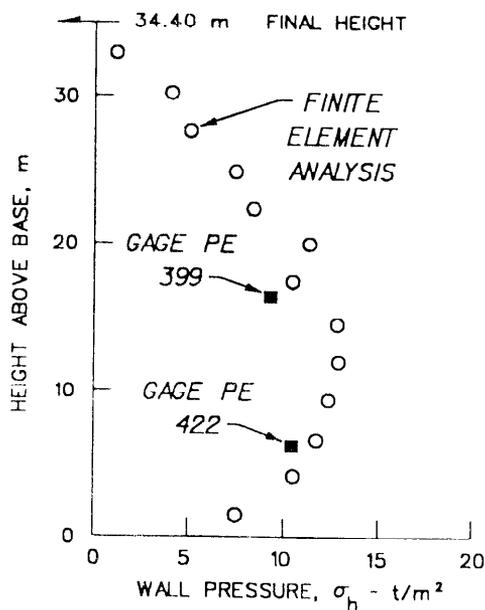
a. Cross section with finite element mesh and earth pressure cells



b. Results of nonlinear finite element calculations and gage PE 399 - during backfilling

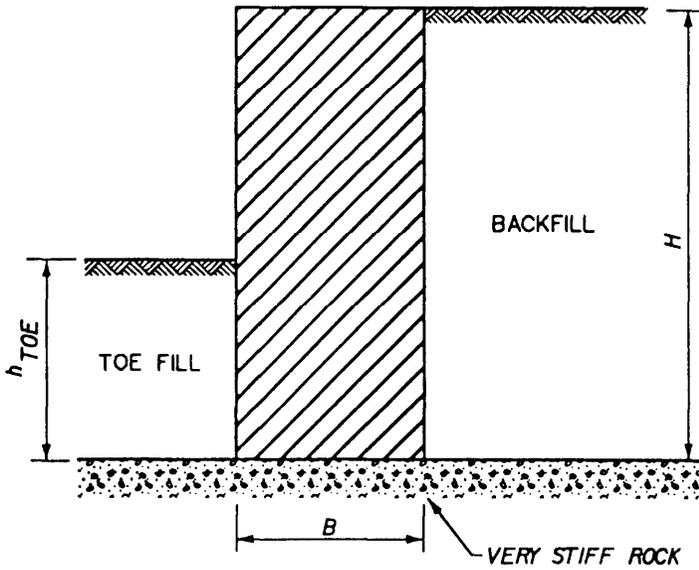


c. Results of nonlinear finite element calculations and gage PE 422 - during backfilling

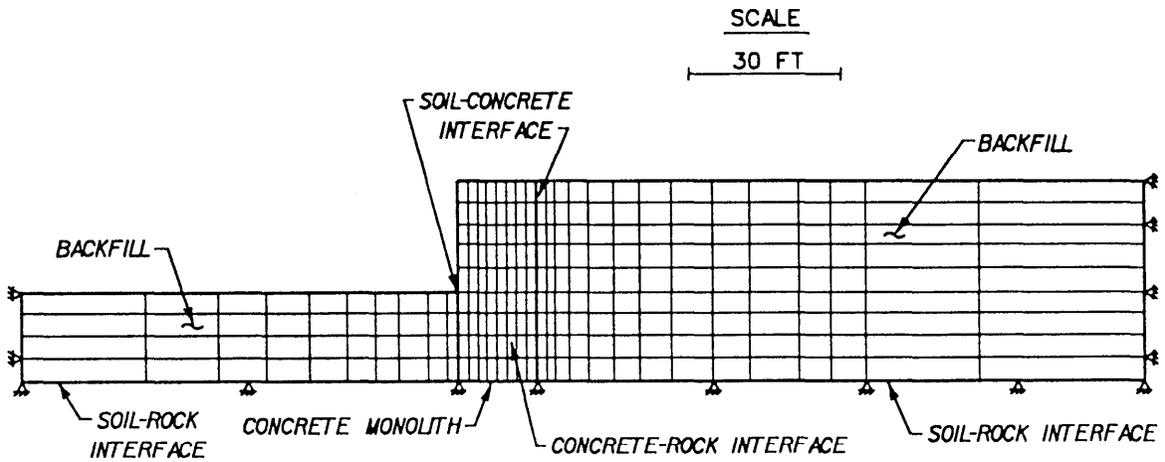


d. Results of nonlinear finite element calculations and gages PE 399 and PE 422 - after backfilling

Figure 15. Study by Roth, Lee, and Crandall (1979)



*a. Rectangular hypothetical structure used in the backfill placement analysis*



*b. Finite element mesh used to model a rectangular hypothetical structure with additional backfill beyond the toe*

*Figure 16. Study by Ebeling, Duncan, and Clough (1989)*

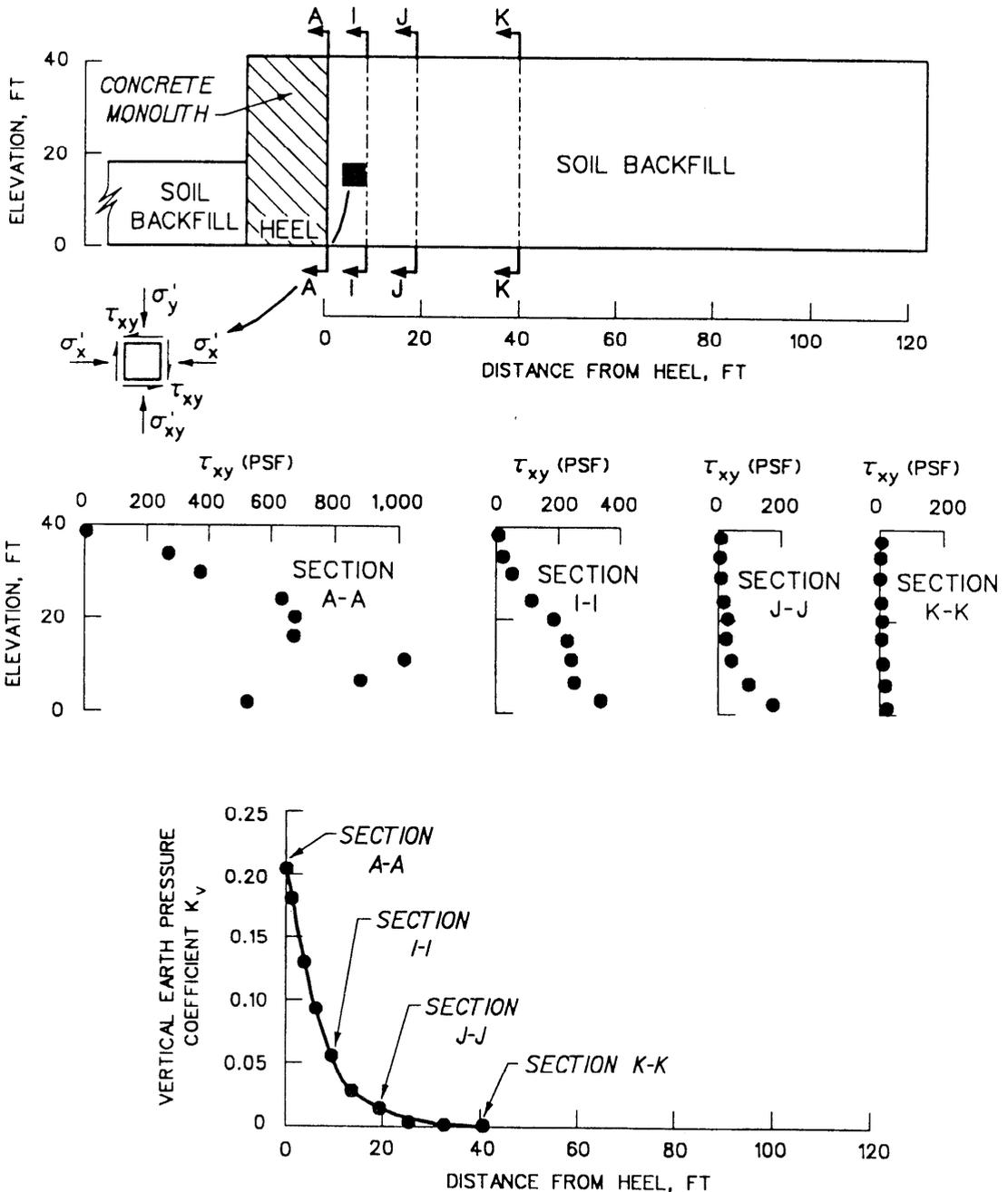


Figure 17. Variation of distribution of shear stress and vertical earth pressure coefficient with distance from the heel of the wall (Ebeling, Duncan, and Clough 1989)

H=40 FT, B=16 FT FOR ALL CASES, FOUNDATION - HARD ROCK  
 BACKFILL AND TOE FILL - SAND:  $\gamma$ -135 LB/FT,  $\phi$ -39°,  $\delta$ -31°(MAXIMUM),  $k_a$ -0.21,  $k_o$ -0.51

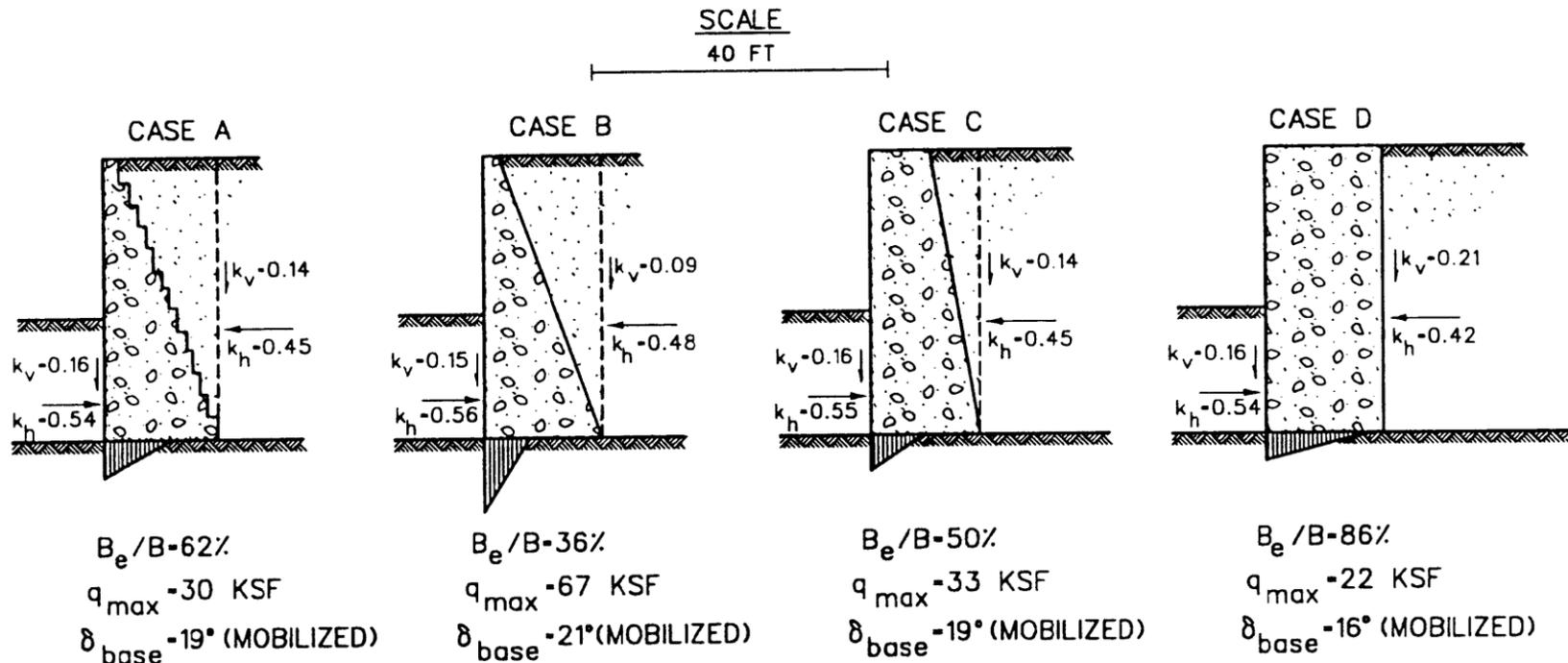
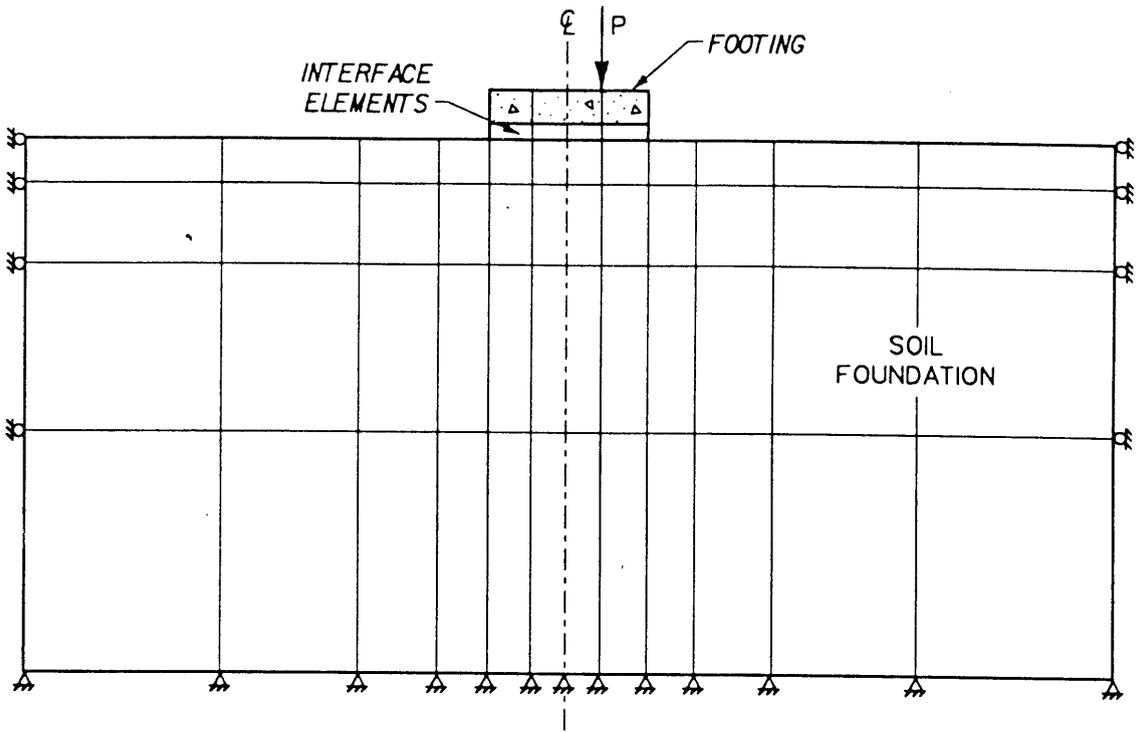
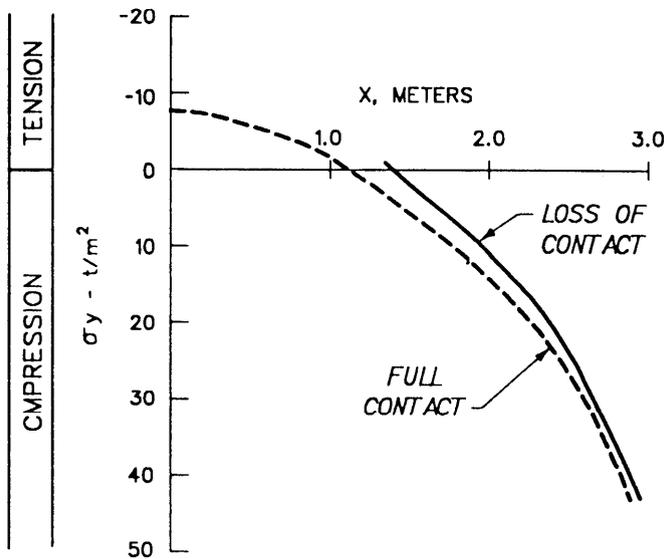


Figure 18. Results of finite element analyses of four retaining walls on rock (Ebeling, Duncan, and Clough 1989)



a. Finite element idealization of footing-soil continuum



b. Contact stress distribution

Figure 19. Study by Desai, Mistry, and Patel (1985)

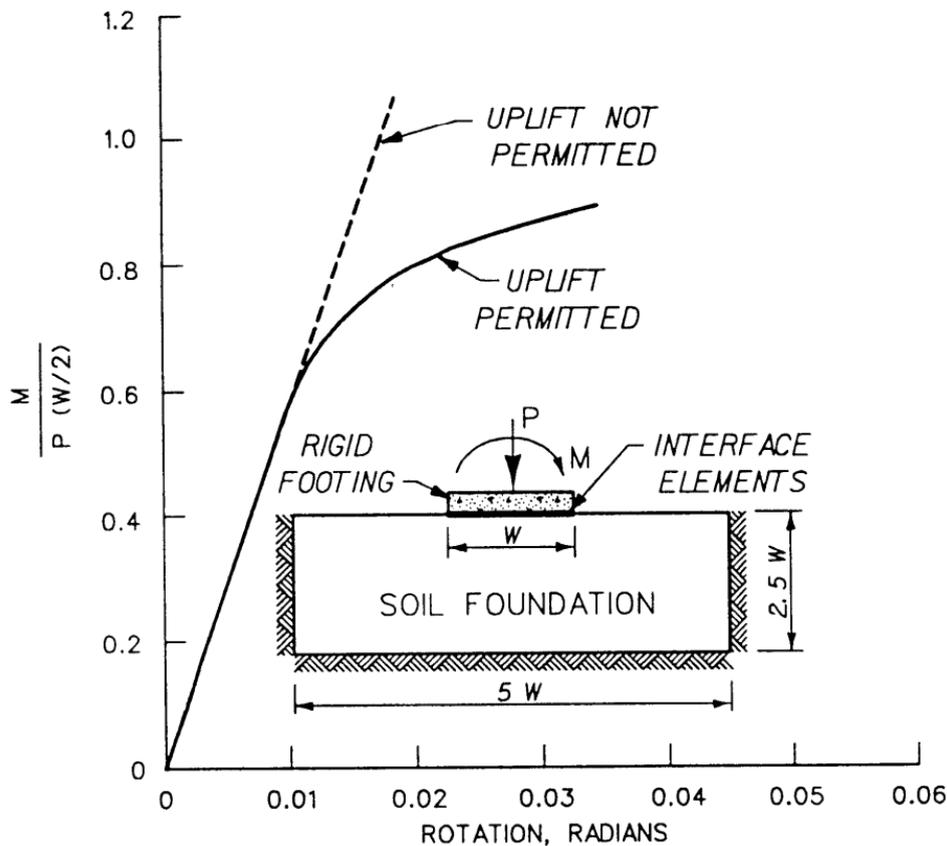
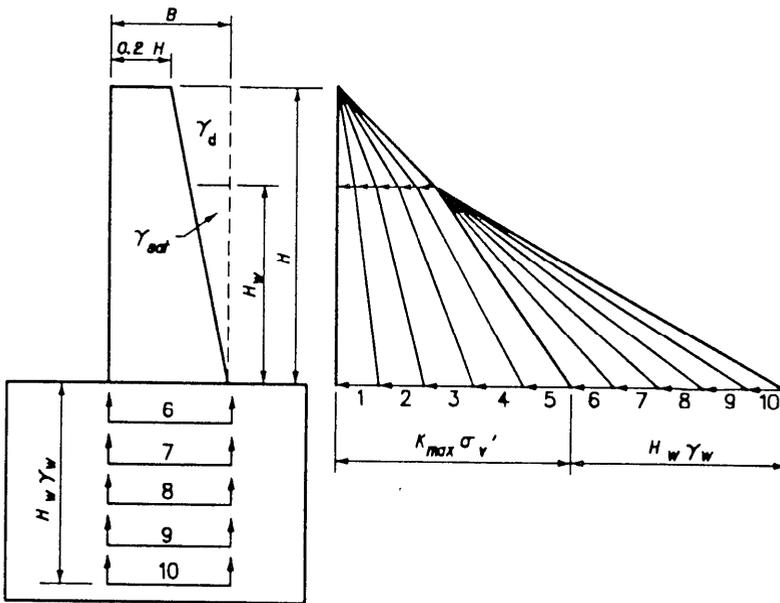
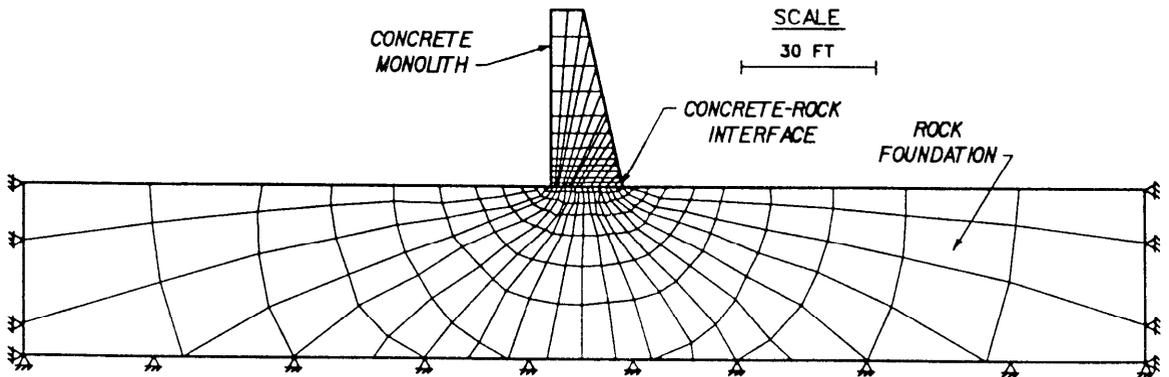


Figure 20. Moment-rotation relationship for footing (Herrmann 1978)



a. Incremental pressure applications used in following load analysis



b. Fine finite element mesh used to model the base case hypothetical structure

Figure 21. Study by Ebeling et al. (1988)

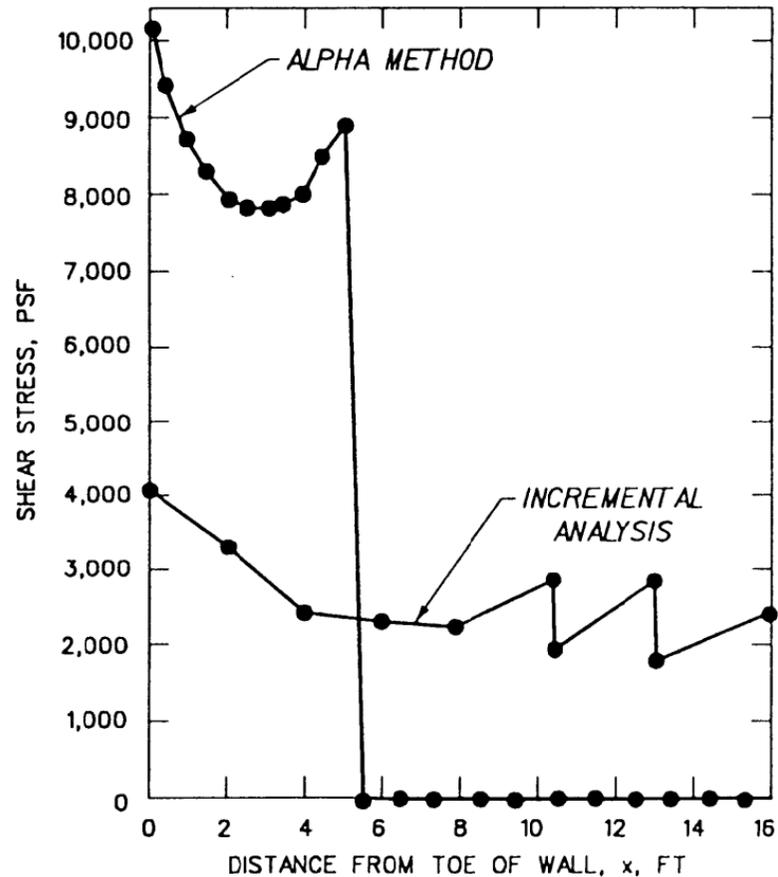
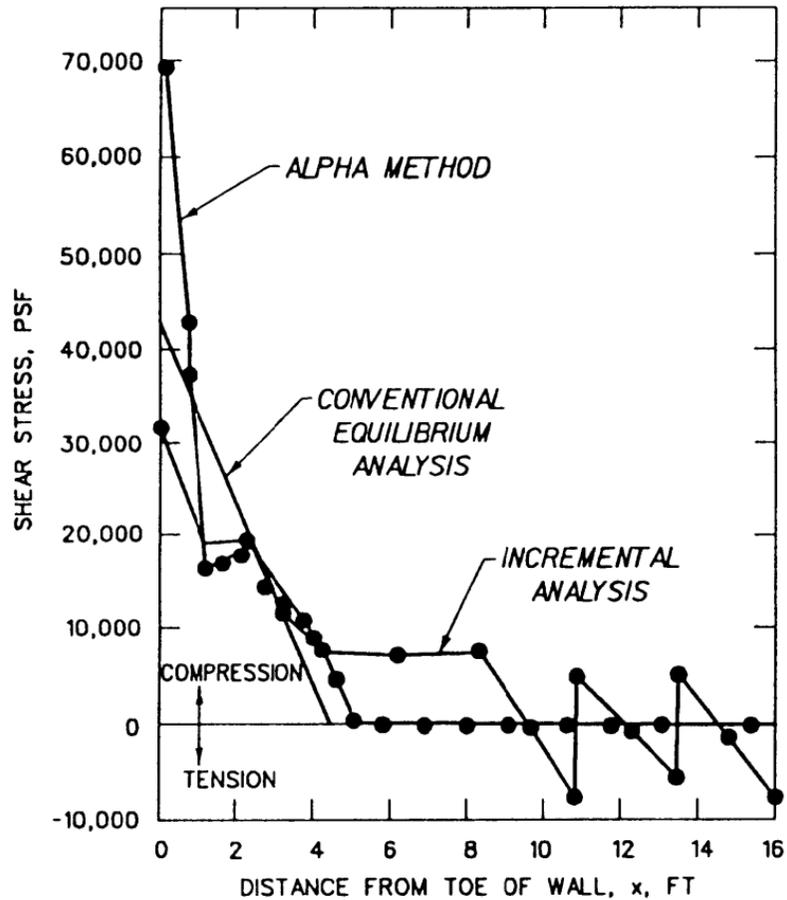


Figure 22. Stress distributions along the base for 0.5  $\sigma_v$  load (Ebeling et al. 1988)

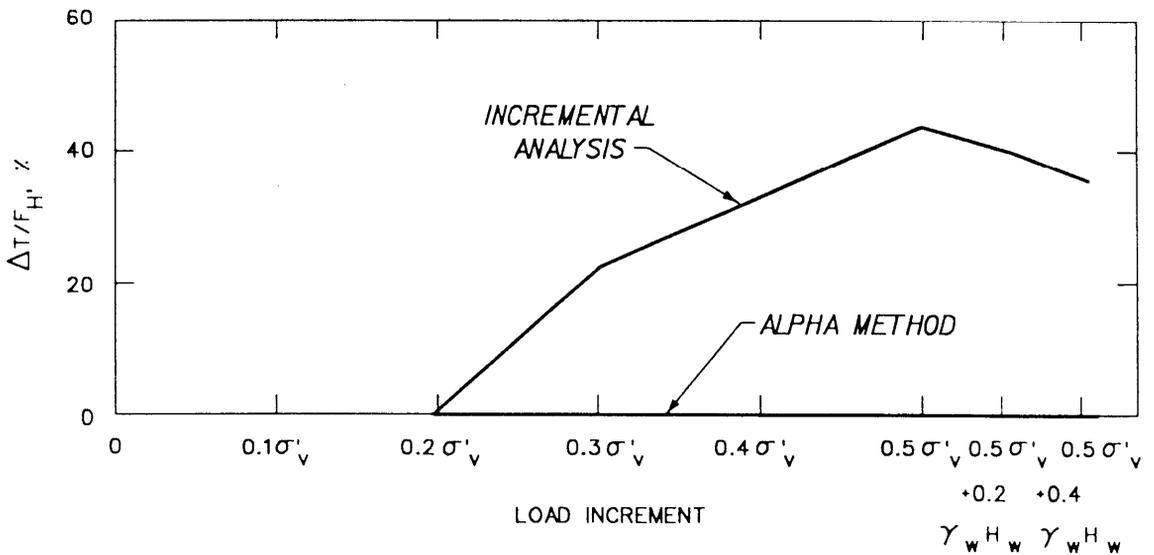
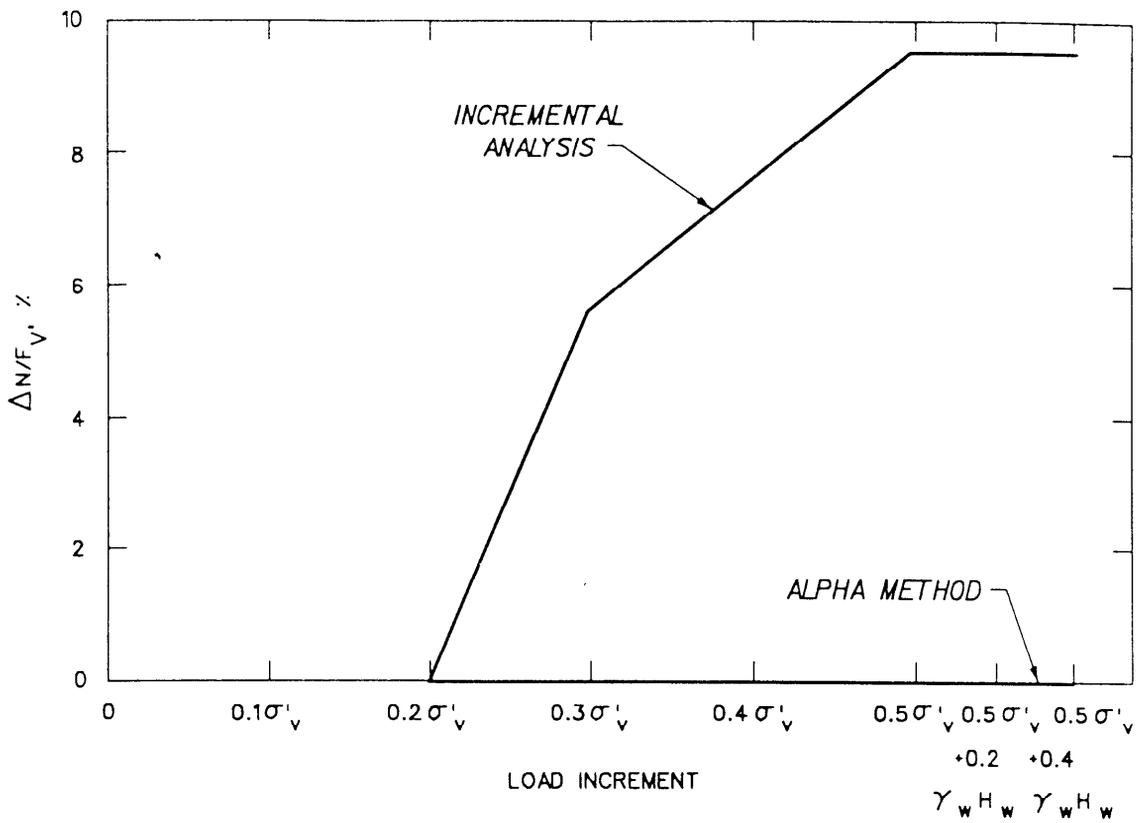
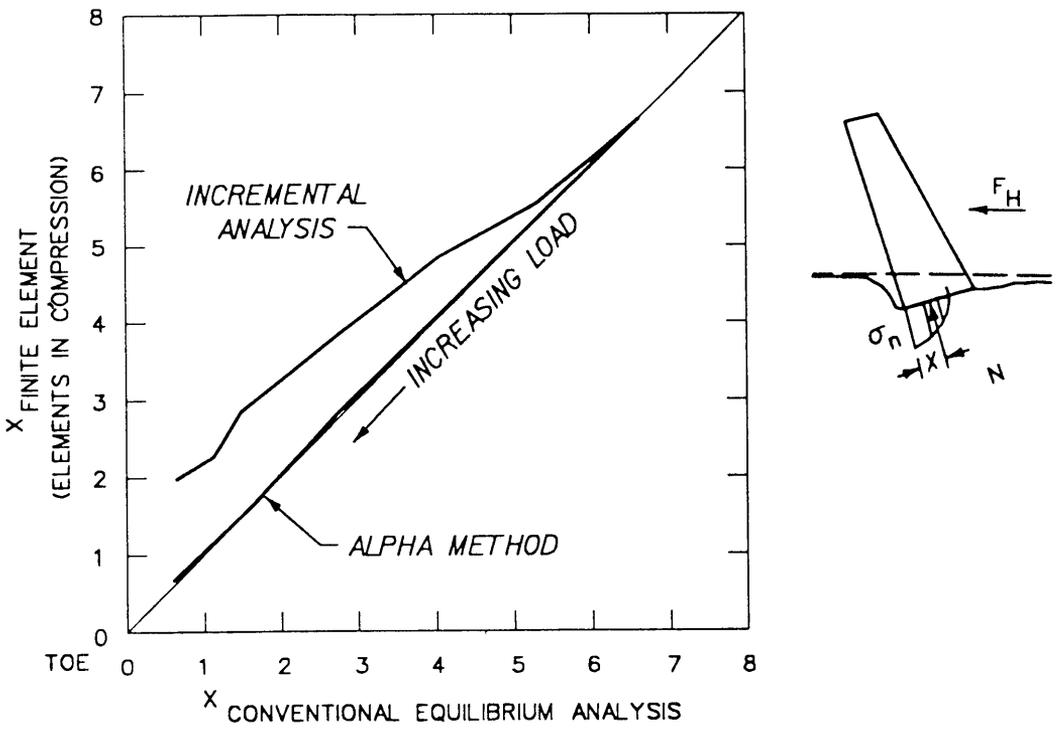
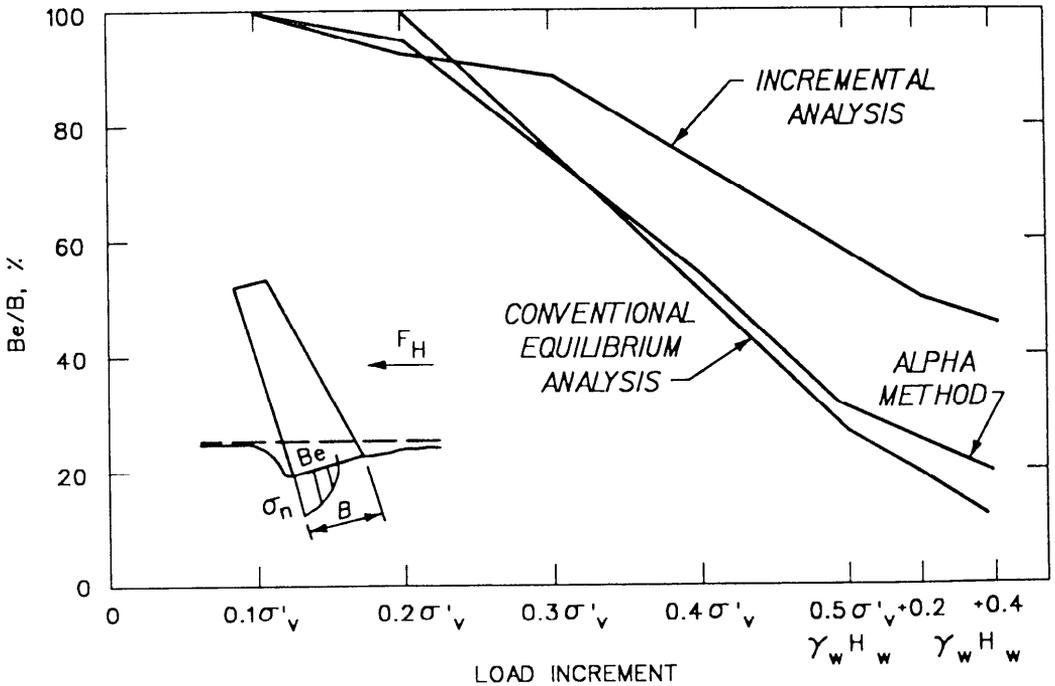


Figure 23. Development of overshoot forces with lateral load (Ebeling et al. 1988)

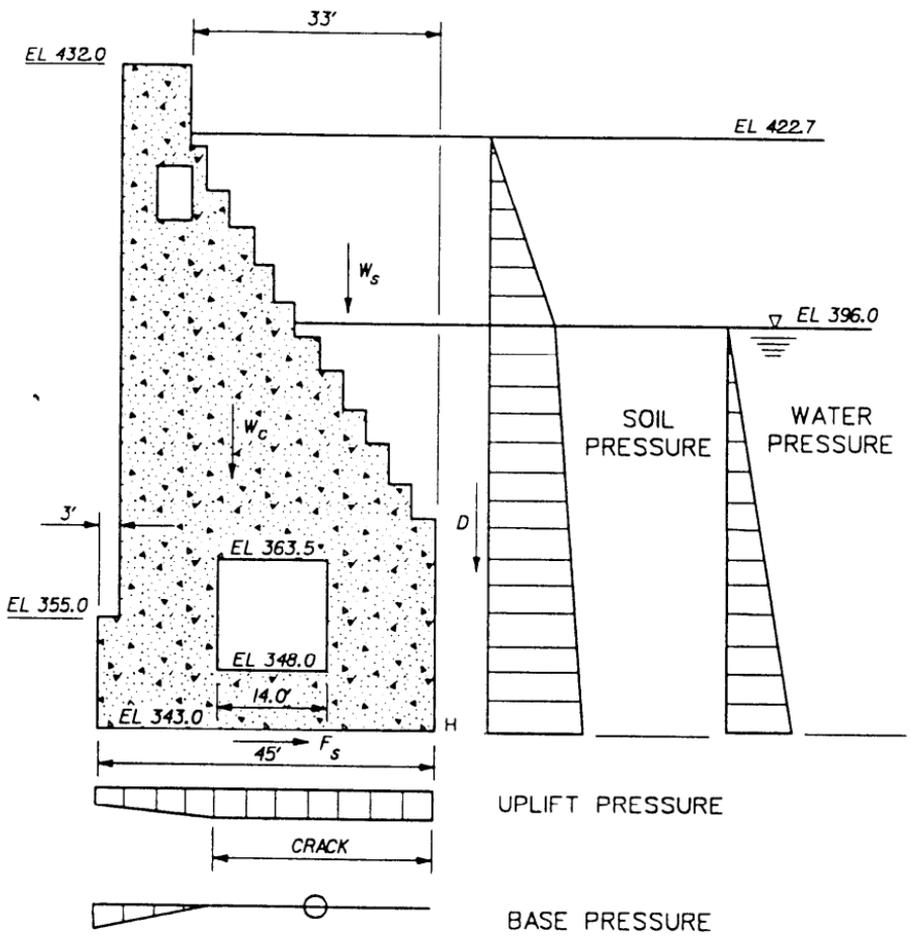


a. Location of resultant normal force as computed using finite element and conventional equilibrium methods

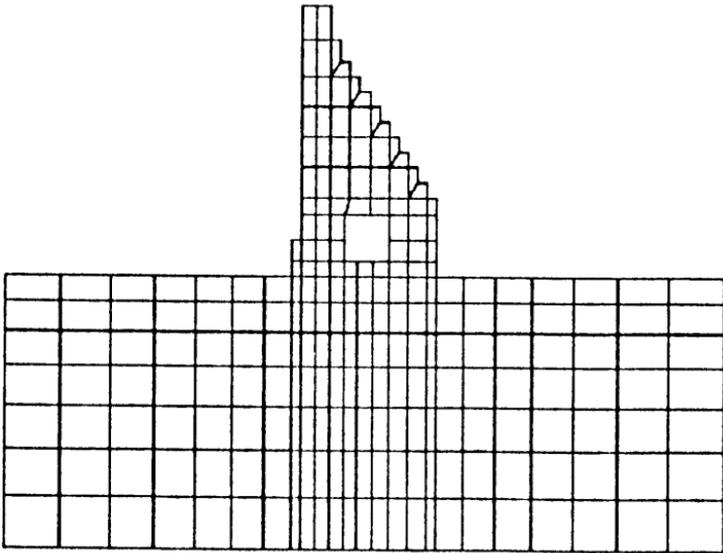


b. Variation of  $B_e/B$  with lateral load (Ebeling et al. 1988)

Figure 24. Accuracy assessment of base separation model



a. Applied load combination



b. Finite element mesh

Figure 25. Analysis of monolith 7e, Lock No. 27  
(Headquarters, US Army Corps of Engineers 1990)